

Fila A

1. $\int x \sqrt{x+1} dx$

Sea $u = x+1$. Entonces $du = dx$ y $x = u-1$.

Por lo tanto:

$$\int x \sqrt{x+1} dx = \int (u-1) u^{1/2} du = \int (u^{3/2} - u^{1/2}) du$$
$$= \frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2} + C = \frac{2}{5} (x+1)^{5/2} - \frac{2}{3} (x+1)^{3/2} + C$$

2. $\int \frac{e^x dx}{(2e^x+3)^{3/2}}$

Sea $u = 2e^x+3$. Entonces

$$du = 2e^x dx \Rightarrow \frac{du}{2} = e^x dx.$$

Por lo tanto:

$$\int \frac{e^x dx}{(2e^x+3)^{3/2}} = \frac{1}{2} \int \frac{du}{(u)^{3/2}} = \frac{1}{2} \int u^{-3/2} du$$

$$= \frac{1}{2} \frac{u^{-3/2+1}}{-3/2+1} + C = \frac{1}{2} \frac{u^{-1/2}}{-1/2} + C = -\frac{1}{\sqrt{u}} + C$$

$$= -\frac{1}{\sqrt{2e^x+3}} + C$$

Fila A.

$$3. \int \frac{\cos x \, dx}{2+3 \operatorname{sen} x}$$

Sea $u = 2 + 3 \operatorname{sen} x$. Entonces

$$du = 3 \cos x \, dx$$

$$\frac{du}{3} = \cos x \, dx. \quad \text{Por lo tanto:}$$

$$\int \frac{\cos x \, dx}{2+3 \operatorname{sen} x} = \frac{1}{3} \int \frac{du}{u} = \frac{1}{3} \ln |u| + C \\ = \frac{1}{3} \ln |2+3 \operatorname{sen} x| + C$$

$$4. \int \frac{x \, dx}{x^2+6x+10} = \int \frac{x \, dx}{(x^2+6x+9)+1} = \int \frac{x \, dx}{(x+3)^2+1}$$

Sea $u = x+3$. Entonces $du = dx$ y $x = u-3$.

Por lo tanto:

$$\int \frac{x \, dx}{x^2+6x+10} = \int \frac{(u-3) \, du}{u^2+1} = \int \frac{u}{u^2+1} \, du - 3 \int \frac{du}{u^2+1}$$

$$= \frac{1}{2} \ln |u^2+1| - 3 \arctan(u) + C$$

$$= \frac{1}{2} \ln |(x+3)^2+1| - 3 \arctan(x+3) + C$$

$$= \frac{1}{2} \ln |x^2+6x+10| - 3 \arctan(x+3) + C$$

Fila B

$$1. \int \frac{\text{Sen } x}{1+5 \cos x} dx = \frac{1}{5} \int \frac{du}{u} = -\frac{1}{5} \ln |u| + C$$

Sea $u = 1 + 5 \cos x$
 $du = -5 \text{ Sen } x dx$
 $\frac{du}{-5} = \text{Sen } x dx$

$$= -\frac{1}{5} \ln |1 + 5 \cos x| + C$$

$$2. \int \frac{e^x dx}{(3e^x + 2)^{5/2}} = \frac{1}{3} \int \frac{du}{u^{5/2}} = \frac{1}{3} \int u^{-5/2} du$$

Sea $u = 3e^x + 2$
 $du = 3e^x dx$
 $\frac{du}{3} = e^x dx$

$$= \frac{1}{3} \cdot \frac{u^{-5/2+1}}{-5/2+1} + C$$

$$= \frac{1}{3} \frac{u^{-3/2}}{-3/2} + C = -\frac{2}{9} u^{-3/2} + C$$

$$= -\frac{2}{9} (3e^x + 2)^{-3/2} + C$$

$$3. \int x \sqrt{x+2} dx = \int (u-2) u^{1/2} dx = \int (u^{3/2} - 2u^{1/2}) du$$

Sea $u = x+2 \Rightarrow x = u-2$
 $du = dx$

$$= \int u^{3/2} du - 2 \int u^{1/2} du$$

$$= \frac{2}{5} u^{5/2} - 2 \cdot \frac{2}{3} u^{3/2} + C$$

$$= \frac{2}{5} (x+2)^{5/2} - \frac{4}{3} (x+2)^{3/2} + C$$

Fila B

$$4. \int \frac{x}{x^2+4x+5} dx = \int \frac{x dx}{(x^2+4x+4)+1} = \int \frac{x dx}{(x+2)^2+1}$$

Sea $u = x+2$. Entonces $du = dx$ \wedge $x = u-2$.

Por lo tanto:

$$\int \frac{x}{x^2+4x+5} dx = \int \frac{(u-2) du}{u^2+1} = \int \frac{u}{u^2+1} du - 2 \int \frac{du}{u^2+1}$$

$$= \frac{1}{2} \ln|u^2+1| - 2 \arctan(u) + C$$

$$= \frac{1}{2} \ln|(x+2)^2+1| - 2 \arctan(x+2) + C$$

$$= \frac{1}{2} \ln|x^2+4x+5| - 2 \arctan(x+2) + C$$