A Solución del tercer parcial de cálculo II

Octubre 20 de 2018

Hora: 13:00 – 14:50

1. Puntos de corte: $f(x) = 5-x$, $g(x) = x^2-7x+10$

$5-x = x^2-7x+10 \Leftrightarrow x^2-6x+5 = 0$

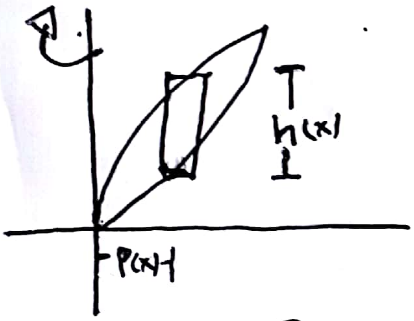
$\Leftrightarrow (x-1)(x-5) = 0 \Leftrightarrow x=1,\ x=5$

$$A = \int_1^5 [(5-x)-(x^2-7x+10)]\,dx = \int_1^5 -x^2+6x-5\,dx$$

$$= \left[\frac{-x^3}{3}+3x^2-5x\right]_1^5 = \left(-\frac{125}{3}+75-25\right)-\left(\frac{1}{3}+3-5\right)$$

$$= \frac{25}{3}-\left(-7/3\right) = \frac{32}{3}$$

2: Por el método de capas.



$p(x) = x \quad h(x) = (-x^2+4x)-x^2$

$h(x) = -2x^2+4x$

Puntos de corte:

$-x^2+4x = x^2 \Leftrightarrow 2x^2-4x=0$

$\Leftrightarrow 2x(x-2) = 0 \Leftrightarrow x=0,\ x=2$

$$\Rightarrow V = 2\pi\int_0^2 p(x)h(x)\,dx = 2\pi\int_0^2 x(-2x^2+4x)\,dx$$

$$= 2\pi\int_0^2 -2x^3+4x^2\,dx = 2\pi\left[\frac{4}{3}x^3-\frac{x^4}{2}\right]_0^2 = \frac{8}{3}\cdot 2\pi$$

$$V = \frac{16}{3}\pi$$

3. $l = \int_1^3 \sqrt{1 + [y'(x)]^2}\, dx$, $y(x) = \frac{1}{8}x^4 + \frac{1}{4x^2}$

$y'(x) = \frac{1}{2}x^3 - \frac{1}{2x^3} = \frac{x^6 - 1}{2x^3}$

$[y'(x)]^2 = \frac{x^{12} - 2x^6 + 1}{4x^6}$

$[y'(x)]^2 + 1 = \frac{x^{12} - 2x^6 + 1}{4x^6} + 1 = \frac{x^{12} + 2x^6 + 1}{4x^6} = \frac{(x^6+1)^2}{4x^6}$

$\Rightarrow l = \int_1^3 \frac{x^6+1}{2x^3}\, dx = \int_1^3 \frac{x^3}{2} + \frac{1}{2x^3}\, dx = \left[\frac{x^4}{8} - \frac{1}{4x^2}\right]_1^3$

$\Rightarrow l = \frac{92}{9}$

4. $\int_e^{+\infty} \frac{dx}{x \ln^4 x} = \lim_{b \to +\infty} \int_e^b \frac{dx}{x \ln^4 x} = \lim_{b \to +\infty} \left[-\frac{1}{3\ln^3 x}\right]_e^b$

$= \lim_{b \to +\infty} \left[-\frac{1}{3\ln^3 b} + \frac{1}{3\ln^3 e}\right] = 0 + \frac{1}{3} = \frac{1}{3}$

B Solución del tercer parcial de cálculo II

octubre 20 de 2018

Hora 13:00 – 14:50

1. Puntos de corte:

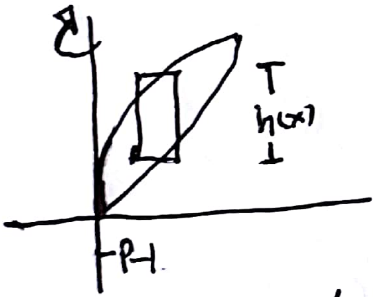
$$x-2 = x^2-7x+10 \Leftrightarrow x^2-8x+12=0$$

$$\Leftrightarrow (x-2)(x-6)=0 \Leftrightarrow x=2,\ x=6$$

$$A=\int_2^6 [(x-2)-(x^2-7x+10)]\,dx = \int_2^6 (-x^2+8x-12)\,dx$$

$$=\left[-\frac{x^3}{3}+4x^2-12x\right]_2^6 = \frac{32}{3}$$

2. Por el metodo de capas.



Puntos de corte:

$$-x^2+2x = x^2 \Leftrightarrow 2x^2-2x=0$$

$$2x(x-1)=0 \Leftrightarrow x=0 \vee x=1$$

$$p(x)=x,\ h(x)=(-x^2+2x)-x^2=-2x^2+2x$$

$$V=2\pi\int_0^1 x(-2x^2+2x)\,dx = 2\pi\int_0^1 -2x^3+2x^2\,dx$$

$$=2\pi\left[-\frac{1}{2}x^4+\frac{2}{3}x^3\right]_0^1 = 2\pi\left[\left(-\frac{1}{2}+\frac{2}{3}\right)-0\right]=\frac{\pi}{3}$$

3. Siguiendo exactamente la solución de la fila A tenemos

$$l=\left[\frac{x^4}{8}-\frac{1}{4x^2}\right]_1^2 = \left[\left(2-\frac{1}{16}\right)-\left(\frac{1}{8}-\frac{1}{4}\right)\right]$$

$$=\frac{31}{16}+\frac{1}{8}=\frac{33}{16}$$

4. $\int_{e}^{+\infty} \frac{dx}{x\ln^3 x} = \lim_{b\to+\infty} \int_{e}^{b} \frac{dx}{x\ln^3 x} = \lim_{b\to+\infty} \left[-\frac{1}{2\ln^2 x}\right]_{e}^{b}$

$= \lim_{b\to+\infty} \left[-\frac{1}{2\ln^2 b} + \frac{1}{2\ln^2 e}\right] = 0 + \frac{1}{2}$

$= \frac{1}{2}$

A

Solución del tercer parcial de Cálculo 2

20/10/2018.

Hora: 15:10–17.

I. ¿Puntos de corte? $x-1=x^2-5x+4 \Rightarrow x^2-6x+5=0$

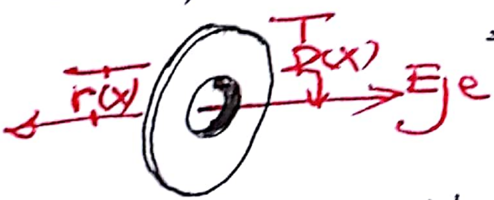
$\Rightarrow (x-1)(x-5)=0 \Rightarrow x=1,\ x=5.$

$$A=\int_1^5\left[(x-1)-(x^2-5x+4)\right]dx=\int_1^5(-x^2+6x-5)\,dx=\frac{32}{3}$$

II. Por el método de las arandelas.

¿Puntos de corte? $3-x=x^2+1 \Rightarrow x^2+x-2=0 \Rightarrow (x+2)(x-1)=0$

$\Rightarrow x=-2,\ x=1.$ $\quad R(x)=3-x,\ r(x)=x^2+1 \Rightarrow V=\pi\int_{-2}^{1}\left[(3-x)^2-(x^2+1)^2\right]dx \Rightarrow$



$$\Rightarrow V=\pi\int_{-2}^{1}(9-6x+x^2-x^4-2x^2-1)\,dx \Rightarrow$$

$$V=\pi\int_{-2}^{1}(8-6x-x^2-x^4)\,dx=\frac{117\pi}{5}$$

III. $y'=\frac{2}{3}\left(\frac{3}{2}\right)(x^2+1)^{1/2}(2x) \Rightarrow (y')^2=4x^2(x^2+1)=4x^4+4x^2 \Rightarrow$

$(y')^2+1=4x^4+4x^2+1=(2x^2+1)^2. \Rightarrow \int_0^1\sqrt{1+(y')^2}\,dx$

$$=\int_0^1\sqrt{(2x^2+1)^2}\,dx=\int_0^1(2x^2+1)\,dx=\left(\frac{2}{3}x^3+x\right)\Big|_0^1=\frac{5}{3}$$

IV. $\lim_{b\to\infty}\int_2^b\frac{3\,dx}{x^2+4}=\lim_{b\to\infty}\left(\frac{3}{2}\right)\arctan\left(\frac{x}{2}\right)\Big|_2^b$

$$=\lim_{b\to\infty}\left(\frac{3}{2}\right)\left[\arctan\left(\frac{b}{2}\right)-\arctan(1)\right]=\left(\frac{3}{2}\right)\left(\frac{\pi}{2}-\frac{\pi}{4}\right)$$

$$=\left(\frac{3}{2}\right)\left(\frac{\pi}{4}\right)=\frac{3\pi}{8}$$

D

B

Solución del tercer parcial de Cálculo 2.

20/10/2018

Hora: 15:10 - 17.

I. ¿Puntos de corte? $4-x=x^2-5x+4 \Rightarrow x^2-4x=0 \Rightarrow$

$x(x-4)=0 \Rightarrow x=0,\ x=4.$

$$A=\int_0^4([4-x]-[x^2-5x+4])dx=\int_0^4(-x^2+4x)dx=\left(-\frac{x^3}{3}+2x^2\right)\Big|_0^4$$

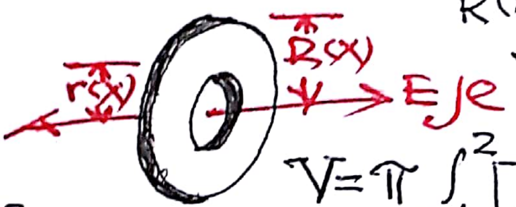
$$=-\frac{64}{3}+32=\frac{32}{3}$$

II. $x+3=x^2+1 \Rightarrow x^2-x-2=0 \Rightarrow (x-2)(x+1)=0$

$x=-1,\ x=2.$

Método de las arandelas.

$R(x)=x+3,\ r(x)=x^2+1$



$$V=\pi\int_{-1}^{2}[(x+3)^2-(x^2+1)^2]dx$$

$$V=\pi\int_{-1}^{2}[(x^2+6x+9)-(x^4+2x^2+1)]dx \Rightarrow$$

$$V=\pi\int_{-1}^{2}(-x^4-x^2+6x+8)dx=\pi\left(-\frac{x^5}{5}-\frac{x^3}{3}+3x^2+8x\right)\Big|_{-1}^{2}=\frac{117\pi}{5}$$

III. $y'=\frac{2}{3}\left(\frac{3}{2}\right)(x^2+1)^{1/2}(2x) \Rightarrow (y')^2=4x^2(x^2+1)=4x^4+4x^2 \Rightarrow$

$(y')^2+1=4x^4+4x^2+1=(2x^2+1)^2 \Rightarrow$

$$\int_0^2\sqrt{1+(y')^2}\,dx=\int_0^2\sqrt{(2x^2+1)^2}\,dx=\int_0^2(2x^2+1)dx=\frac{2}{3}x^3+x\Big|_0^2$$

$$=\frac{22}{3}$$

IV $\lim_{b\to\infty}\int_{\sqrt{3}}^{b}\frac{3dx}{x^2+1}=\lim_{b\to\infty}(3)\arctan(x)\Big|_{\sqrt{3}}^{b} \Rightarrow$

$\lim_{b\to\infty}(3)\left[\arctan\left(\frac{b}{2}\right)-\arctan(\sqrt{3})\right]=(3)\left(\frac{\pi}{2}-\frac{\pi}{3}\right)$

$$=(3)\left(\frac{\pi}{6}\right)=\frac{\pi}{2}$$