Fila A

A

1. $u = x^3 + 1$

$du = 3x^2\,dx$

$\frac{du}{3} = x^2\,dx$

$\int 18x^2\sqrt{x^3+1}\,dx = \int 6u^{1/2}\,du$

$= 6 \cdot \frac{2}{3} u^{3/2} + C.$

$= 4(x^3+3)^{3/2} + C.$

$\Rightarrow \int_0^2 18x^2\sqrt{x^3+1}\,dx = \left[4(x^3+7)^{3/2}\right]_0^2 = \left[4(27) - 4\right]$

$= 108 - 4 = 104$

2. $\int x^3 \ln x\,dx$

$u = \ln x \qquad dv = x^3\,dx$

$du = \frac{1}{x}dx \qquad v = \frac{1}{4}x^4$

$\Rightarrow \int x^3 \ln x\,dx = \frac{1}{4}x^4 \ln x - \frac{1}{4}\int x^3\,dx$

$= \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C.$

3. $\int \frac{dx}{x^3\sqrt{x^2-4}}$

$x = 2\sec\theta$

$dx = 2\sec\theta\tan\theta\,d\theta$

$\Rightarrow \int \frac{dx}{x^3\sqrt{x^2-4}} = \int \frac{2\sec\theta\tan\theta\,d\theta}{8\sec^3\theta\sqrt{4(\sec^2\theta - 1)}}.$

$= \frac{1}{8}\int \frac{\sec\theta\tan\theta}{\sec^3\theta\tan\theta}\,d\theta = \frac{1}{8}\int \frac{1}{\sec^2\theta}\,d\theta$

A

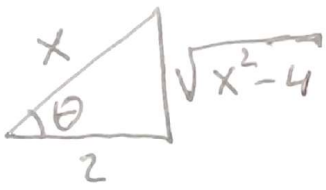
$$= \frac{1}{8}\int \cos^2\theta \, d\theta = \frac{1}{8}\int \frac{1+\cos 2\theta}{2} \, d\theta$$

$$= \frac{1}{8}\left[\frac{1}{2}\theta + \frac{1}{4}\operatorname{sen} 2\theta\right] + C$$

$$= \frac{1}{16}\theta + \frac{1}{32}\,2\operatorname{sen}\theta\cos\theta + C$$

$$= \frac{1}{16}\theta + \frac{1}{16}\operatorname{sen}\theta\cos\theta + C$$

$$\sec\theta = \frac{x}{2}, \quad \cos\theta = \frac{2}{x}$$



$$\operatorname{sen}\theta = \frac{\sqrt{x^2-4}}{x}$$

$$\Rightarrow \int \frac{dx}{x^3\sqrt{x^2-4}} = \frac{1}{16}\operatorname{arcsec}\left(\frac{x}{2}\right) + \frac{1}{16}\frac{\sqrt{x^2-4}}{x}\cdot\frac{2}{x} + C.$$

$$= \frac{1}{16}\operatorname{arcsec}\left(\frac{x}{2}\right) + \frac{1}{8}\frac{\sqrt{x^2-4}}{x^2} + C$$

4. $$\int \frac{2x^2\,dx}{(x+1)(x^2+1)}$$

$$\frac{2x^2}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1)+(x+1)(Bx+C)}{(x+1)(x^2+1)}$$

$$2x^2 = Ax^2 + A + Bx^2 + Cx + Bx + C$$

$$2x^2 = (A+B)x^2 + (C+B)x + (A+C)$$

$$\Rightarrow \begin{cases} A+B=2 \\ C+B=0 \\ A+C=0 \end{cases} \Rightarrow \begin{array}{l} A+B=2 \\ A-B=0 \\ \hline 2A=2 \Rightarrow A=1 \end{array}$$

$B = 1$

$C = -1$

A

$$\Rightarrow \int \frac{2x^2\,dx}{(x+1)(x^2+1)} = \int \frac{dx}{x+1} + \int \frac{x-1}{x^2+1}\,dx$$

$$= \int \frac{dx}{x+1} + \int \frac{x}{x^2+1}\,dx - \int \frac{dx}{x^2+1}$$

$$= \ln|x+1| + \frac{1}{2}\ln|x^2+1| - \arctan x + C$$

Fila B. B

1. $\int_0^2 9x^2\sqrt{x^3+1}\,dx$

$u = x^3+1$

$du = 3x^2dx$

$\frac{du}{3} = x^2dx$

$\int 9x^2\sqrt{x^3+1}\,dx = 9/3\int u^{1/2}du$

$= \cancel{3}\cdot\frac{2}{\cancel{3}}u^{3/2} + C$

$= 2(x^3+1)^{3/2} + C$

$\Rightarrow \int_0^2 9x^2\sqrt{x^3+1}\,dx = \left[2(x^3+1)^{3/2}\right]_0^2$

$= [54-2] = 52$

2. $\int x^2\ln x\,dx$

$u = \ln x \qquad du = x^2dx$

$du = \frac{1}{x}dx \qquad u = \frac{1}{3}x^3$

$\Rightarrow \int x^2\ln x\,dx = \frac{1}{3}x^3\ln x - \frac{1}{3}\int x^2dx$

$= \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C$

3. $\int \frac{dx}{x^3\sqrt{x^2-9}}$

$x = 3\sec\theta$

$dx = 3\sec\theta\tan\theta\,d\theta$

$\Rightarrow \int \frac{dx}{x^3\sqrt{x^2-9}} = \int \frac{3\sec\theta\tan\theta\,d\theta}{27\sec^3\theta\sqrt{9(\sec^2\theta-1)}}$

$$= \frac{1}{27}\int \frac{d\theta}{\sec^2\theta} = \frac{1}{27}\int \cos^2\theta\, d\theta$$

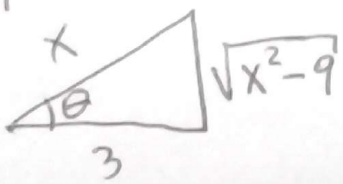
$$= \frac{1}{27}\int \frac{1+\cos 2\theta}{2} d\theta = \frac{1}{54}\left[\int d\theta + \int \cos 2\theta\, d\theta\right]$$

$$= \frac{1}{54}\left[\theta + \operatorname{sen}\theta\cos\theta\right] + C$$

$$= \frac{1}{54}\left[\operatorname{arcsec}\left(\frac{x}{3}\right) + \frac{\sqrt{x^2-9}}{x}\cdot\frac{3}{x}\right] + C$$

$$= \frac{1}{54}\left[\operatorname{arcsec}\left(\frac{x}{3}\right) + 3\frac{\sqrt{x^2-9}}{x^2}\right] + C$$

$\sec\theta = \frac{x}{3}$ $\quad \cos\theta = \frac{3}{x}$



$\operatorname{sen}\theta = \frac{\sqrt{x^2-9}}{x}$

4. $\int \frac{2x^2\, dx}{(x-1)(x^2+1)}$

$$\frac{2x^2}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1)+(Bx+C)(x-1)}{(x-1)(x^2+1)}$$

$$= \frac{Ax^2+A+Bx^2-Bx+Cx-C}{(x-1)(x^2+1)}$$

$$= \frac{(A+B)x^2+(C-B)x+(A-C)}{(x-1)(x^2+1)}$$

$$\Rightarrow \begin{cases} A+B=2 \\ C-B=0 \\ A-C=0 \end{cases} \Rightarrow \begin{array}{r} A+B=2 \\ A-B=0 \\ \hline 2A=2 \Rightarrow A=1 \end{array}$$

$B = 1$

$C = 1$

$$\Rightarrow \int \frac{2x^2\,dx}{(x-1)(x^2+1)} = \int \frac{dx}{x-1} + \int \frac{x}{x^2+1}\,dx + \int \frac{dx}{x^2+1}$$

$$= \ln|x-1| + \frac{1}{2}\ln|x^2+1| + \arctan(x) + C.$$