Fila A.

1. Pontos de corte:

$$x^2+1 = 3x^2-1 \Rightarrow 2x^2-2=0$$

$$\Rightarrow 2(x^2-1)=0 \Rightarrow 2(x+1)(x-1)=0$$

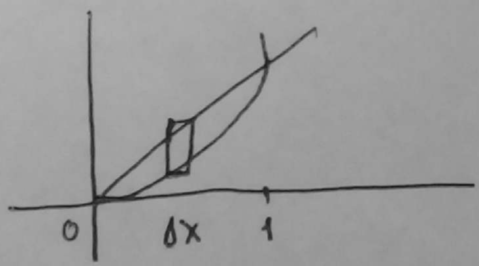
$$x=1 \;,\; x=-1.$$

$$A=\int_{-1}^{1}(x^2+1)-(3x^2-1)\,dx = \int_{-1}^{1}-2x^2+2\,dx$$

$$A=\left[-\frac{2}{3}x^3+2x\right]_{-1}^{1} = \left(-\frac{2}{3}+2\right)-\left(\frac{2}{3}-2\right) = -\frac{4}{3}+4 = \frac{8}{3}$$

$$\Rightarrow A = 8/3$$

2. Por arandelas:



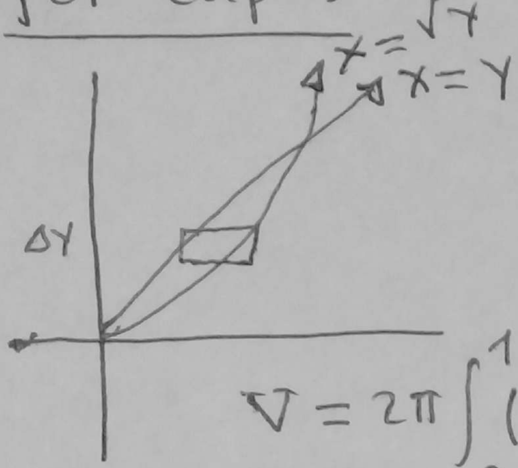
$$R(x) = x$$
$$r(x) = x^2$$

$$V = \pi\int_0^1 (x^2-x^4)\,dx = \pi\left[\frac{x^3}{3}-\frac{x^4}{5}\right]_0^1$$

$$= \pi\left[\left(\frac{1}{3}-\frac{1}{5}\right)\right] = \frac{2\pi}{15}$$

Fila A

2. Por capas



$$p(y) = y$$
$$h(y) = \sqrt{y} - y$$
$$V = 2\pi \int_0^1 y(\sqrt{y} - y)\, dy$$
$$V = 2\pi \int_0^1 (y^{3/2} - y^2)\, dy = 2\pi \left[\frac{2}{5} y^{5/2} - \frac{y^3}{3}\right]_0^1$$
$$V = 2\pi \left[\frac{2}{5} - \frac{1}{3}\right] = \frac{2\pi}{15} \Rightarrow V = \frac{2\pi}{15}$$

3. $y = \frac{1}{6} x^3 + \frac{1}{2x} \Rightarrow y' = \frac{1}{2}\left(x^2 - \frac{1}{x^2}\right)$

$$1 + [y']^2 = \frac{1}{4}\left(x^2 + \frac{1}{x^2}\right)^2$$
$$\Rightarrow \ell = \int_1^3 \sqrt{1 + [y']^2}\, dx = \int_1^3 \frac{1}{2}\left(x^2 + \frac{1}{x^2}\right) dx$$
$$\ell = \left[\frac{1}{2}\left(\frac{x^3}{3} - \frac{1}{x}\right)\right]_1^3 = \frac{1}{2}\left(9 - \frac{1}{3}\right) - \frac{1}{2}\left(\frac{1}{3} - 1\right)$$
$$\ell = \frac{14}{3}$$

Fila A

4. $$\int_{-\infty}^{+\infty} \frac{3x\,dx}{(x^2+1)^2} = \int_{-\infty}^{0} \frac{3x\,dx}{(x^2+1)^2} + \int_{0}^{+\infty} \frac{3x\,dx}{(x^2+1)^2}$$

$$= \lim_{a \to -\infty} \int_{a}^{0} \frac{3x\,dx}{(x^2+1)^2} + \lim_{b \to +\infty} \int_{0}^{b} \frac{3x\,dx}{(x^2+1)^2}$$

$$\Rightarrow \int_{-\infty}^{+\infty} \frac{3x\,dx}{(x^2+1)^2} = \lim_{a \to -\infty} \left[-\frac{3}{2} \frac{1}{(x^2+1)} \right]_a^0 + \lim_{b \to +\infty} \left[-\frac{3}{2} \frac{1}{(x^2+1)} \right]_0^b$$

$$= -\frac{3}{2} + \lim_{a \to -\infty} \frac{3}{2(a^2+1)} + \lim_{b \to +\infty} -\frac{3}{2} \cdot \frac{1}{(b^2+1)} + \frac{3}{2}$$

$$= -\frac{3}{2} + 0 + 0 + \frac{3}{2} = 0$$

Fila B.

1. Puntos de corte:

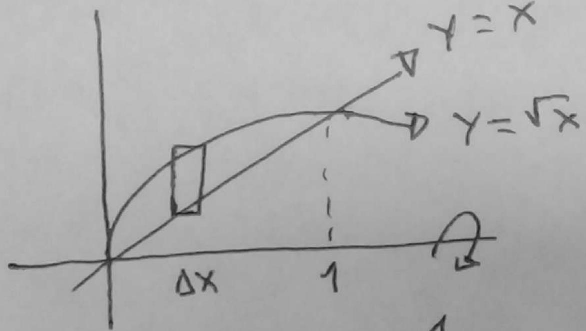
$$2x^2-4=x^2 \Rightarrow x^2-4=0 \Rightarrow x=2,\ x=-2$$

$$A=\int_{-2}^{2}(x^2-(2x^2-4))\,dx=\int_{-2}^{2}(-x^2+4)\,dx$$

$$A=\left[-\frac{x^3}{3}+4x\right]_{-2}^{2}=\left(-\frac{8}{3}+8\right)-\left(\frac{8}{3}-8\right)$$

$$=-\frac{16}{3}+16=\frac{32}{3} \Rightarrow A=\frac{32}{3}$$

2. Por arandelas



$R(x)=\sqrt{x}$

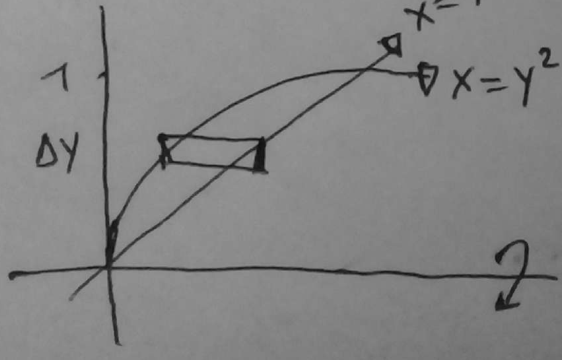
$r(x)=x$

$$V=\pi\int_0^1 (x-x^2)\,dx$$

$$\Rightarrow V=\pi\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1=\pi\left(\frac{1}{2}-\frac{1}{3}\right)=\frac{\pi}{6}$$

$$\Rightarrow V=\pi/6$$

Por capas



$p(y)=y$

$h(y)=y-y^2$

$$V=2\pi\int_0^1 y(y-y^2)\,dy$$

$$=2\pi\left[y^3/3-y^4/4\right]_0^1$$

$$\Rightarrow V=2\pi/12 \Rightarrow V=\pi/6$$

Fila B

3. $Y = \frac{1}{6}x^3 + \frac{1}{2x} \Rightarrow Y' = \frac{1}{2}\left(x^2 - \frac{1}{x^2}\right)$

$\Rightarrow 1 + [Y']^2 = \frac{1}{4}\left(x^2 + \frac{1}{x^2}\right)^2$

$\Rightarrow l = \int_1^2 \sqrt{1 + [Y']^2}\, dx = \int_1^2 \frac{1}{2}\left(x^2 + \frac{1}{x^2}\right) dx$

$l = \left[\frac{1}{2}\left(\frac{x^3}{3} - \frac{1}{x}\right)\right]_1^2 = \frac{1}{2}\left(\frac{8}{3} - \frac{1}{2}\right) - \frac{1}{2}\left(\frac{1}{3} - 1\right)$

$l = \frac{13}{12} + \frac{1}{3} = \frac{17}{12} \Rightarrow l = \frac{17}{12}$

4. $\int_{-\infty}^{+\infty} \frac{4x\,dx}{(x^2+1)^2} = \int_{-\infty}^{0} \frac{4x\,dx}{(x^2+1)^2} + \int_{0}^{+\infty} \frac{4x\,dx}{(x^2+1)^2}$

$= \lim_{a \to -\infty} \int_a^0 \frac{4x\,dx}{(x^2+1)^2} + \lim_{b \to +\infty} \int_0^b \frac{4x\,dx}{(x^2+1)^2}$

$\Rightarrow \int_{-\infty}^{+\infty} \frac{4x\,dx}{(x^2+1)^2} = \lim_{a \to -\infty} \left[\frac{-2}{x^2+1}\right]_a^0 + \lim_{b \to +\infty} \left[\frac{-2}{x^2+1}\right]_0^b$

$= -2 + \lim_{a \to -\infty} \frac{2}{a^2+1} + \lim_{b \to +\infty} \frac{-2}{b^2+1} + 2$

$= -2 + 0 + 0 + 2 = 0$