Solución Parcial 3A-2018-2

1. $$y^{(4)} - y''' + y'' - y' = 12x + 4 + 10e^{2x}.$$

Hallemos la solución de la ec. homogénea asociada:

$$y^{(4)} - y''' + y'' - y' = 0$$

$$m^4 - m^3 + m^2 - m = 0$$

$$m(m^3 - m^2 + m - 1) = 0$$

$$m\left[m^2(m-1) + 1\cdot(m-1)\right] = 0$$

$$m(m-1)(m^2+1) = 0$$

$$m = 0, \qquad m = 1, \qquad m = 0 \pm 1i$$

$$\boxed{y_c(x) = c_1 + c_2 e^{x} + c_3 \cos x + c_4 \operatorname{sen} x.}$$

Hallemos la solución particular:

$$y_p = y_{p_1} + y_{p_2}$$

$$y_p = Ax + B + Ce^{2x} \qquad (\text{NO})$$

Puesto que hay duplicaciones con respecto a la homogénea,

$$y_p = Ax^2 + Bx + Ce^{2x}.$$

$$y_p' = 2Ax + B + 2Ce^{2x}$$

$$y_p'' = 2A + 4Ce^{2x}$$

$$y_p''' = 8Ce^{2x}$$

$$y_p^{(4)} = 16Ce^{2x}$$

$$y_p^{(4)} - y_p''' + y_p'' - y_p' = 12x + 4 + 10e^{2x}$$

$$16Ce^{2x} - 8Ce^{2x} + (2A + 4Ce^{2x}) - (2Ax + B + 2Ce^{2x}) = 12x + 4 + 10e^{2x}$$

$$-2Ax + 10Ce^{2x} + 2A - B = 12x + 4 + 10e^{2x}$$

$$-2A = 12 \qquad 10C = 10 \qquad 2A - B = 4$$

$$\boxed{A = -6} \qquad \boxed{C = 1} \qquad B = 2A - 4 = -12 - 4 \; ; \; \boxed{B = -16}$$

Por tanto $\boxed{y_p(x) = -6x^2 - 16x + e^{2x}}$

En consecuencia, $y(x) = y_c(x) + y_p(x)$

$$\boxed{y(x) = c_1 + c_2e^{x} + c_3\cos x + c_4 \operatorname{sen} x}$$

2. $(x^2+1)y'' - 2xy' + 2y = 12(x^2+1)^2$, $y_1(x) = x$.

$$y'' - \frac{2x}{x^2+1}\cdot y' + \frac{2}{x^2+1}y = \underbrace{12(x^2+1)}_{f(x)}$$

$$P(x) = -\frac{2x}{x^2+1}.$$

a) $$e^{-\int P(x)dx} = e^{\int \frac{2x}{x^2+1}dx} = e^{\ln(x^2+1)} = x^2+1$$

$$y_2(x) = x\int \frac{x^2+1}{x^2}dx = x\left(\int dx + \int \frac{1}{x^2}dx\right) = x\left(x - \frac{1}{x}\right)$$

$\boxed{y_2(x) = x^2-1}$ $\Rightarrow$ $\boxed{y_c(x) = c_1 x + c_2(x^2-1)}$

b) $$W = \begin{vmatrix} x & x^2-1 \\ 1 & 2x \end{vmatrix} = 2x^2 - x^2 + 1 = x^2+1$$

$\boxed{W = x^2+1}$

$$W_1 = \begin{vmatrix} 0 & x^2-1 \\ 12(x^2+1) & 2x \end{vmatrix} = -12(x^2+1)(x^2-1)$$

$\boxed{W_1 = -12(x^2+1)(x^2-1)}$

$$W_2 = \begin{vmatrix} x & 0 \\ 1 & 12(x^2+1) \end{vmatrix} = 12x(x^2+1)$$

$\boxed{W_2 = 12x(x^2+1)}$

$$u_1 = \int \frac{W_1}{W}\,dx = -12\int \frac{\cancel{(x^2+1)}(x^2-1)}{\cancel{x^2+1}}\,dx = -12\int (x^2-1)\,dx = -12\left(\frac{x^3}{3} - x\right)$$

$$\boxed{u_1 = 12x - 4x^3}$$

$$u_2 = \int \frac{W_2}{W}\,dx = 12\int \frac{x\cancel{(x^2+1)}}{\cancel{x^2+1}}\,dx = 12\int x\,dx = 12\,\frac{x^2}{2} = 6x^2$$

$$\boxed{u_2 = 6x^2}$$

$$y_p = y_1u_1 + y_2u_2 = x(12x - 4x^3) + (x^2-1)(6x^2)$$

$$= 12x^2 - 4x^4 + 6x^4 - 6x^2$$

$$\boxed{y_p = 6x^2 + 2x^4}$$

$$\boxed{y(x) = c_1x + c_2(x^2-1) + 6x^2 + 2x^4}$$

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3. $x^2 y'' + 5xy' + 5y = 0$

Sea $y = x^m$. Entonces $y' = m x^{m-1}$, $y'' = m(m-1) x^{m-2}$. Sustituyendo

$$x^2 \left[m(m-1) x^{m-2} \right] + 5x \left(m x^{m-1} \right) + 5 x^m = 0$$

$$x^m \left(m^2 - m + 5m + 5 \right) = 0$$

$$m^2 + 4m + 5 = 0$$

$$m = \frac{-4 \pm \sqrt{16 - 4(1)(5)}}{2} = \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm 2i}{2}$$

$$\boxed{m = \underset{\alpha}{-2} \pm \underset{\beta}{1} i}$$

$$\boxed{y(x) = c_1 x^{-2} \cos(\ln x) + c_2 x^{-2} \operatorname{sen}(\ln x)}$$