1. $(x^2-1)y'' - 2xy' + 2y = (x^2-1)^2$

Se sabe que $y_1(x) = x$ es una solución de la ec. homogénea asociada.

a) $y'' \underbrace{- \frac{2x}{x^2-1}}_{p(x)} y' + \frac{2}{x^2-1} y = \underbrace{x^2-1}_{f(x)}$

$$e^{-\int p(x)dx} = e^{-\int -\frac{2x}{x^2-1}dx} = e^{\int \frac{2x}{x^2-1}dx} = e^{\ln(x^2-1)} = x^2-1 \quad (0.5)$$

$$y_2(x) = x \int \frac{x^2-1}{x^2} dx = x \left[\int \left(1 - \frac{1}{x^2}\right) dx \right] = x\left(x + \frac{1}{x}\right)$$

$$\boxed{y_2(x) = x^2+1} \quad (0.5)$$

b) $y_c(x) = c_1 x + c_2(x^2+1)$.

$$W = \begin{vmatrix} x & x^2+1 \\ 1 & 2x \end{vmatrix} = 2x^2 - x^2 - 1 = x^2-1 \; ; \boxed{W = x^2-1} \quad (0.3)$$

$$W_1 = \begin{vmatrix} 0 & x^2+1 \\ x^2-1 & 2x \end{vmatrix} = -(x^2-1)(x^2+1) \; ; \boxed{W_1 = -(x^2-1)(x^2+1)} \quad (0.3)$$

$$W_2 = \begin{vmatrix} x & 0 \\ 1 & x^2-1 \end{vmatrix} = x(x^2-1) \; ; \boxed{W_2 = x(x^2-1)} \quad (0.3)$$

$$u_1 = -\int (x^2+1)\,dx = -\frac{x^3}{3} - x \; ; \boxed{u_1 = -\frac{x^3}{3} - x} \quad (0.3)$$

$$u_2 = \int x\,dx = \frac{x^2}{2} \; ; \boxed{u_2 = \frac{x^2}{2}} \quad (0.3)$$

$$y_p = x\left(-\frac{x^3}{3} - x\right) + (x^2+1)\left(\frac{x^2}{2}\right) = -\frac{x^4}{3} - x^2 + \frac{x^4}{2} + \frac{x^2}{2} = \frac{1}{6}x^4 - \frac{1}{2}x^2$$

$$\boxed{y_p = \frac{1}{6}x^4 - \frac{1}{2}x^2} \quad (0.4) \qquad \boxed{y = c_1 x + c_2(x^2+1) + \frac{1}{6}x^4 - \frac{1}{2}x^2} \quad (0.1)$$

2. $W = 1\,N$ $\quad f(t) = \text{sen}\, t$

$s = 2$ mts $\quad m = ?$

$x'(0) = 0$ $\quad k = ?$ $\quad (0.2)$

$x(0) = 1$

$\beta = \frac{1}{5}$

$$m = \frac{W}{g} = \frac{1}{10}\ \text{kg}. \qquad (0.1)$$

$$k = \frac{W}{s} = \frac{1}{2} \qquad (0.1)$$

$$\begin{cases} \frac{1}{10}x''(t) + \frac{1}{5}x'(t) + \frac{1}{2}x(t) = \text{sen}\, t \\ x(0) = 1 \quad , \quad x'(0) = 0 \end{cases}$$

$$\begin{cases} x'' + 2x' + 5x = 10\,\text{sen}\, t \\ x(0) = 1, \quad x'(0) = 0. \end{cases} \qquad (0.2)$$

Hallemos $x_c(t)$: $x'' + 2x' + 5x = 0$.

$$m^2 + 2m + 5 = 0$$

$$m^2 + 2m + 1 = -4$$

$$(m+1)^2 = -4$$

$$m + 1 = \pm 2i \qquad (0.4)$$

$$\boxed{m = -1 \pm 2i}$$

$$x_c(t) = c_1 e^{-t}\cos 2t + c_2 e^{-t}\,\text{sen}\, 2t.$$

Hallemos $x_p(t)$: $\quad x_p(t) = A\cos t + B\,\text{sen}\,t \qquad (0.1)$

$$x_p'(t) = -A\,\text{sen}\,t + B\cos t$$

$$x_p''(t) = -A\cos t - B\,\text{sen}\,t$$

sustituyendo en la ED:

$$-A\cos t - B\,\text{sen}\,t + 2(-A\,\text{sen}\,t + B\cos t) + 5(A\cos t + B\,\text{sen}\,t) = 10\,\text{sen}\,t$$

$$(4A + 2B)\cos t + (-2A + 4B)\,\text{sen}\,t = 10\,\text{sen}\,t$$

$$\begin{cases} 4A + 2B = 0 \\ -2A + 4B = 10 \end{cases} \quad ; \quad 2B = -4A \;;\; \boxed{B = -2A}$$

$$-2A + 4(-2A) = 10$$

$$-2A - 8A = 10$$

$$-10A = 10 \;;\; \boxed{A = -1} \;(0.2), \quad \boxed{B = 2} \;(0.2)$$

$$\boxed{x_p(t) = 2\,\text{sen}\,t - \cos t}$$

$$x(t) = c_1 e^{-t}\cos 2t + c_2 e^{-t}\,\text{sen}\,2t + 2\,\text{sen}\,t - \cos t$$

$$1 = x(0) = c_1 - 1 \;;\; \boxed{c_1 = 2} \qquad (0.2)$$

$$x'(t) = -c_1 e^{-t}\cos 2t - 2c_1 e^{-t}\,\text{sen}\,2t - c_2 e^{-t}\,\text{sen}\,2t + 2c_2 e^{-t}\cos 2t$$
$$+ 2\cos t + \text{sen}\,t$$

$$0 = x'(0) = -\cancel{c_1} + 2c_2 + \cancel{2} \;;\; \boxed{c_2 = 0} \qquad (0.2)$$

Por lo tanto la solución del problema de valor inicial es:

$$\boxed{x(t) = 2e^{-t}\cos 2t + 2\,\text{sen}\,t - \cos t} \qquad (0.1)$$