# On $\tau$-factorizations

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**Abstract**

The notion of a $\tau$-factorization or $\tau$-products in the general theory of (non-atomic) factorization on integral domains was defined by Anderson and Frazier, in 2006. The idea is basically to study factorizations by just considering a restriction into the element of interest. As the factorizations into primes, irreducible elements, primary elements, primals, rigids, etc. This was easy to considering the relation $\tau = S \times S$, then they called $x = \lambda x_1 \dots x_n$ a $\tau$-factorization of $x$ if $\lambda$ is a unit and each $x_i$ is in a set $S$, of desired nonzero nonunit elements. Later, they considered any symmetric relation $\tau$ on the set of nonzero nonunit elements of an integral domain, which opens the doors to consider many types of factorizations with unique representations. This theory still in development, basically several Anderson's PhD students followed this theory and Ortiz's Master Students since 2013.

This talk will present some basic structural factorization properties and how this properties can be inherited into this new theory of factorizations. Then will provide some examples to familiarized with the concept. As an example this talk is to provide a flavor of the work done when considering the equivalence relation modulo $n$ on the integers, denoted by $\tau_{(n)}$. Among the results, an introduction to the notions of $\tau_{(n)}$-divisibility, the number of $\tau_{(n)}$-factors, $\tau_{(n)}$-$GCD$ and $\tau_{(n)}$-primes, when $\phi(n) \leq 4$ and some more general.