Departamento de Matemáticas y Estadísticas
Actividad de formación Integral lista de chequeo 2
Cálculo II ANEC
Profesor: Rafael Martínez Solano

Nombre:	**Grupo :**

Tema: Integración por partes

1. Pruebe, Utilizando la integración por partes, que

$$\int \frac{\ln(x)}{x^{\frac{2}{3}}}dx = 3x^{\frac{1}{3}}\left(\ln x\right) - 9x^{\frac{1}{3}} + K$$

   Sugerencia: Tome $u = \ln(x)$

2. Pruebe, Utilizando la integración por partes, que

$$\int \frac{\ln^2(x)}{x^{\frac{1}{3}}}dx = \frac{3}{2}x^{\frac{2}{3}}\ln^2 x - \frac{9}{2}x^{\frac{2}{3}}\ln x + \frac{27}{4}x^{\frac{2}{3}} + K$$

   Sugerencia: Tome $u = \ln^2(x)$

3. Utilizando integración tabular, pruebe que:

$$\int (3x^2 + 2x + 1)e^{2x-1}dx = \frac{3}{2}x^2e^{2x-1} - \frac{1}{2}xe^{2x-1} + \frac{3}{4}e^{2x-1} + K$$

4. Dada $\int x^2(2x+1)^{\frac{2}{3}}dx =$

   (a) Resuelva la integral utilizando integración por sustitución, tomando $u = 2x + 1$

   (b) Resuelva la integral utilizando la integral por partes, $u = x^2; dv = (2x+1)^{\frac{2}{3}}dx$

Tema: integrales utilizando el método de fracciones parciales

1. **Caso I. Factores lineales diferentes.**

   (a)
$$\int \frac{3x+1}{(x+1)(x+2)}dx = 5\ln\left(x+2\right) - 2\ln\left(x+1\right) + K$$

   (b)
$$\int \frac{3x+1}{(x+1)(x-2)}dx = \frac{2}{3}\ln\left(x+1\right) + \frac{7}{3}\ln\left(x-2\right) + K$$

   (c)
$$\int \frac{3x-2}{(x+2)(x-2)}dx = \ln\left(x-2\right) + 2\ln\left(x+2\right) + K$$

   (d)
$$\int \frac{3x-2}{(2x+1)(x-2)}dx = \frac{4}{5}\ln\left(x-2\right) + \frac{7}{10}\ln\left(x+\frac{1}{2}\right) + K$$

2. **Caso II. Factores lineales repetidos**

   (a)
$$\int \frac{x+1}{x(2x+1)^2}dx = \frac{1}{4x+2}\left(2\ln x - 2\ln\left(x+\frac{1}{2}\right) - 4x\ln\left(x+\frac{1}{2}\right) + 4x\ln x + 1\right) + K$$

(b)

$$\int \frac{x+1}{x^2(2x+1)}dx = -\frac{1}{x}\left(x\ln x - x\ln\left(x+\frac{1}{2}\right)+1\right)+K$$

(c)

$$\int \frac{x^2+1}{x^2(2x+1)}dx = -\frac{1}{2x}\left(4x\ln x - 5x\ln\left(x+\frac{1}{2}\right)+2\right)+K$$

(d)

$$\int \frac{x^2+1}{(x-1)^2(2x+1)}dx = \frac{-1}{18\,(x-1)}\left(4\ln(x-1)+5\ln\left(x+\frac{1}{2}\right)-4x\ln(x-1)-5x\ln\left(x+\frac{1}{2}\right)+12\right)+K$$

3. Integro rapidamente funciones de la forma

$$f(x)=(mx+b)^N, N\neq -1;\ \text{así}\ \int_a^b (mx+b)^N dx = \left(\frac{1}{m}\frac{(mx+b)^{N+1}}{N+1}\right)\Bigg|_a^b, N\neq -1$$

(a) $\int_0^2 (4x+1)^{\frac{1}{2}}dx = \frac{13}{3}$

(b) $\int_0^3 (7x+4)^{-\frac{1}{2}}dx = \frac{6}{7}$

(c) $\int_0^2 (27-13x)^{\frac{1}{3}}dx = \frac{60}{13}$

(d) $\int_0^1 (1+7x)^{\frac{1}{3}}dx = \frac{45}{28}$

Integro rapidamente funciones de la forma

$$f(x)=\frac{1}{mx+b}\ \text{así}\ \int_a^b \frac{dx}{mx+b} = \left(\frac{1}{m}\ln(mx+b)\right)\Bigg|_a^b$$

(a) $\int_1^2 \frac{dx}{3x+5} = \frac{1}{3}\ln(11)-\ln(2)$

(b) $\int_2^4 \frac{dx}{2x-1} = \frac{1}{2}\ln(7)-\frac{1}{2}\ln(3)$

(c) $\int_5^8 \frac{dx}{4x+3} = \frac{1}{4}\ln(35)-\frac{1}{4}\ln(23)$

(d) $\int_1^2 \frac{dx}{x+5} = \ln 7-\ln 6$

Integro rapidamente funciones de la forma

$$f(x)=e^{mx+b} así \int_a^b e^{mx+b}dx = \left(\frac{1}{m}e^{mx+b}\right)\Bigg|_a^b$$

(a) $\int_0^1 e^{-3x+2}dx = \frac{1}{3}e^2-\frac{1}{3}e^{-1}$

(b) $\int_0^1 e^{8x-1}dx = \frac{1}{8}e^7-\frac{1}{8}e^{-1}$

(c) $\int_2^6 e^{\frac{3}{2}x+5}dx = \frac{2}{3}e^8\left(e^6-1\right)$

(d) $\int_0^1 e^{8-x}dx = e^8-e^7$

Integral de una función afín $f(x)=mx+b$

$$\int_a^b (mx+b)dx = \left(\frac{mx^2}{2}+bx\right)\Bigg|_a^b$$

(a) $\int_0^2 (4x+1)dx = 10$

(b) $\int_1^2 (5x-2)dx = \frac{11}{2}$

(c) $\int_{-2}^2 (4-3x)dx = 16$

(d) $\int_{-1}^3 (5-2x)dx = 12$

Integral de una función cuadratica $f(x)=a(x-h)^2+k$

$$\int_a^b (a(x-h)^2+k)dx = \left(\frac{a(x-h)^3}{3}+kx\right)\Bigg|_a^b$$

(a) $\int_0^2 \left(2(x+1)^2 - 3\right) dx = \frac{34}{3}$

(b) $\int_1^2 \left(-2(x-3)^2 - 1\right) dx = -\frac{17}{3}$

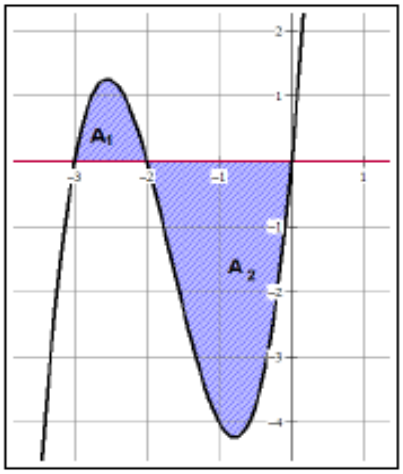
(c) $\int_{-2}^2 \left(3(x-2)^2 + 3\right) dx = 76$

(d) $\int_{-1}^1 \left(-4(x-1)^2 - 1\right) dx = -\frac{38}{3}$

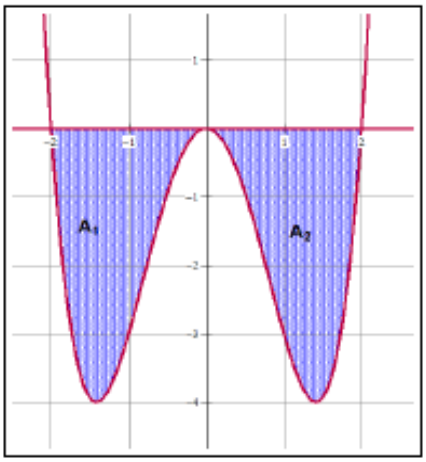
Tema: Expresa el área sombreada en término de integrales

Considere que en cada caso se tiene la gráfica de una función $y = f(x)$

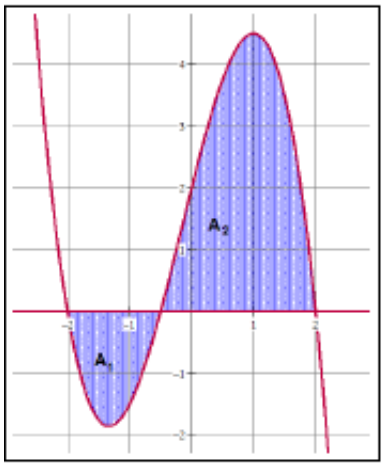
$A_1 + A_2 =$



$A_1 + A_2 =$



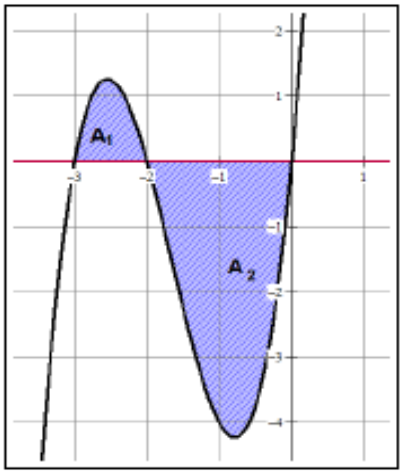
$A_1 + A_2 =$



$A_1 + A_2 =$

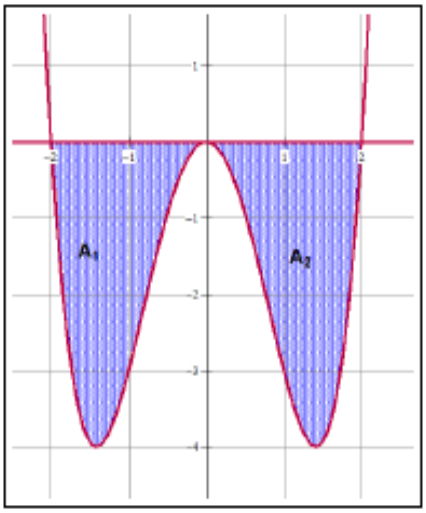
4. Expresa una integral en términos del área encerrada entre la curva y el eje x

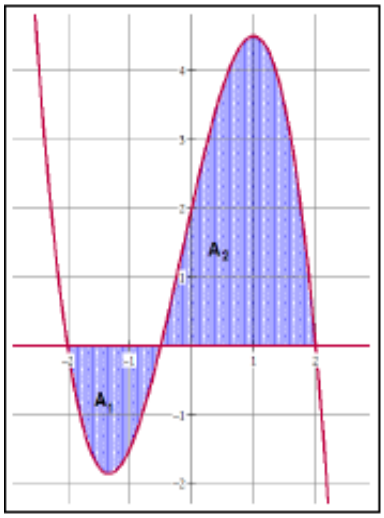
   Considere que en cada caso se tiene la gráfica de una función $y = f(x)$



$\int_{-2}^{2} f(x)dx =$

$\int_{-3}^{0} f(x)dx =$





$\int_{-2}^{2} f(x) =$

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5. resuelve integrales definidas, utilizando procedimientos similares a los estudiados para integrales indefinidas.

   (a) $\int_0^1 x^2 e^{2x+1} dx = \frac{1}{4} e \left(e^2 - 1\right)$

   (b) $\int_1^2 x^2 \ln(x) dx = \frac{8}{3} \ln 2 - \frac{7}{9}$

   (c) $\int_0^2 x \ln^2(x) dx = 2 \ln^2(2) - 2 \ln(2) + 1$

   (d) $\int_0^1 x(2x-3)^4 dx = \frac{179}{30}$

   (e) $\int_0^1 x^2 (7x+1)^{\frac{2}{3}} dx = \frac{84\,837}{75\,460}$