

13 2022-30

Fila A

① $5x^2y'' + 3xy' + 2y = 0 \quad (x > 0)$.

Sol: $y = x^m, y' = mx^{m-1}, y'' = m(m-1)x^{m-2} \Rightarrow 5x^2m(m-1)x^{m-2} + 3mx^{m-1} + 2x^m = 0$.

$5m^2 - 2m + 2 = 0$ c.c. aux.

$m = \frac{2 \pm \sqrt{4 - 4(5)(2)}}{10} = \frac{2 \pm \sqrt{36}i}{10} = \frac{2 \pm 6i}{10} = \frac{1}{5} \pm \frac{3}{5}i$.

Solución general de la ED:
 $y = x^{1/5} (c_1 \cos(\frac{3}{5} \ln x) + c_2 \sin(\frac{3}{5} \ln x))$.

$\alpha = 1/5, \beta = 3/5$.

② $y''' - 4y'' + 3y' = 12xe^x$, cfs: $y_1 = e^x, y_2 = e^{3x}, y_3 = 2$.

$f(x) = 12xe^x$

$W = \begin{vmatrix} e^x & e^{3x} & 2 \\ e^x & 3e^{3x} & 0 \\ e^x & 9e^{3x} & 0 \end{vmatrix} = 2(9e^{4x} - 3e^{4x}) = 12e^{4x}$

$W_1 = \begin{vmatrix} 0 & e^{3x} & 2 \\ 0 & 3e^{3x} & 0 \\ f(x) & 9e^{3x} & 0 \end{vmatrix} = 2(0 - f(x)3e^{3x}) = -6e^{3x}(12xe^x) = -6 \cdot 12 \cdot xe^{4x}$

$W_2 = \begin{vmatrix} e^x & 0 & 2 \\ e^x & 0 & 0 \\ e^x & f(x) & 0 \end{vmatrix} = 2(e^x f(x) - 0) = 2e^x(12xe^x) = 24xe^{2x}$

$W_3 = \begin{vmatrix} e^x & e^{3x} & 0 \\ e^x & 3e^{3x} & 0 \\ e^x & 9e^{3x} & f(x) \end{vmatrix} = f(x) \begin{vmatrix} e^x & e^{3x} \\ e^x & 3e^{3x} \end{vmatrix} = f(x)(3e^{4x} - e^{4x}) = 2e^{4x} f(x) = 2e^{4x}(12xe^x) = 24xe^{5x}$

$u_1 = \frac{W_1}{W} = \frac{-6 \cdot 12xe^{4x}}{12e^{4x}} = -6x \Rightarrow u_1 = -6 \int x dx = -6 \frac{x^2}{2} = -3x^2$

$u_2 = \frac{W_2}{W} = \frac{24xe^{2x}}{12e^{4x}} = 2xe^{-2x} \Rightarrow u_2 = 2 \int xe^{-2x} dx = 2 \left(\frac{e^{-2x}}{-2} x - \int \frac{e^{-2x}}{-2} \cdot 1 dx \right) = 2 \left(-\frac{1}{2}xe^{-2x} + \frac{1}{2} \int e^{-2x} dx \right) = -xe^{-2x} - \frac{1}{2}e^{-2x}$

$u_3 = \frac{W_3}{W} = \frac{24xe^{5x}}{12e^{4x}} = 2xe^x \Rightarrow u_3 = 2 \int xe^x dx = 2(xe^x - \int e^x dx) = 2(xe^x - e^x)$

$y_p = u_1 y_1 + u_2 y_2 + u_3 y_3 = -3x^2 e^x + (-xe^{-2x} - \frac{1}{2}e^{-2x})e^{3x} + 2(xe^x - e^x) \cdot 2$
 $= -3x^2 e^x + (-x - \frac{1}{2})e^x + (4x - 4)e^x = (-3x^2 - x - \frac{1}{2} + 4x - 4)e^x$

$\Rightarrow y_p = (-3x^2 + 3x - \frac{9}{2})e^x$

La solución general de la ED no homogénea es:

$y = (-3x^2 + 3x - \frac{9}{2})e^x + c_1 e^x + c_2 e^{3x} + c_3$