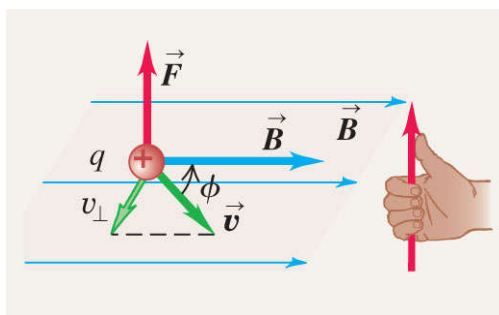


1. **Magnetic forces:** Magnetic interactions are fundamentally interactions between moving charged particles. These interactions are described by the vector magnetic field, denoted by  $\vec{B}$ . A particle with charge  $q$  moving with velocity  $\vec{v}$  in a magnetic field  $\vec{B}$  experiences a force  $\vec{F}$  that is perpendicular to both  $\vec{v}$  and  $\vec{B}$ . The SI unit of magnetic field is the tesla ( $1 \text{ T} = 1 \text{ N/A} \cdot \text{m}$ ). (See Example 27.1.)

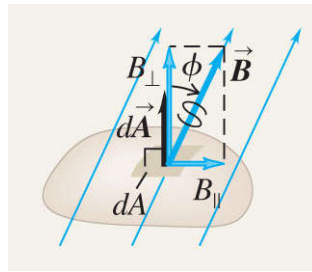
$$\vec{F} = q\vec{v} \times \vec{B} \quad (27.2)$$



2. **Magnetic field lines and flux:** A magnetic field can be represented graphically by magnetic field lines. At each point a magnetic field line is tangent to the direction of  $\vec{B}$  at that point. Where field lines are close together the field magnitude is large, and vice versa. Magnetic flux  $\Phi_B$  through an area is defined in an analogous way to electric flux. The SI unit of magnetic flux is the weber ( $1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2$ ). The net magnetic flux through any closed surface is zero (Gauss's law for magnetism). As a result, magnetic field lines always close on themselves. (See Example 27.2.)

$$\begin{aligned} \Phi_B &= \int B_{\perp} dA \\ &= \int B \cos \phi dA \\ &= \int \vec{B} \cdot d\vec{A} \end{aligned} \quad (27.6)$$

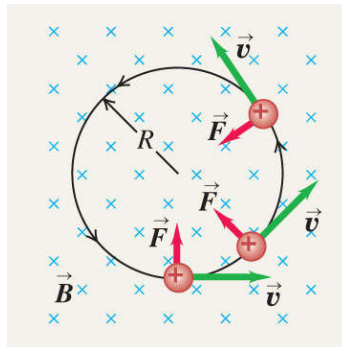
$$\oint \vec{B} \cdot d\vec{A} = 0 \quad (\text{closed surface}) \quad (27.8)$$



3. **Motion in a magnetic field:** The magnetic force is always perpendicular to  $\vec{v}$ ; a particle moving under the action of a magnetic field alone moves with constant speed. In a uniform field, a particle with initial velocity perpendicular to the field moves in a circle with radius  $R$  that depends on the magnetic field strength  $B$  and the particle mass  $m$ , speed  $v$ , and charge  $q$ . (See Examples 27.3 and 27.4.)

Crossed electric and magnetic fields can be used as a velocity selector. The electric and magnetic forces exactly cancel when  $v = E/B$ . (See Examples 27.5 and 27.6.)

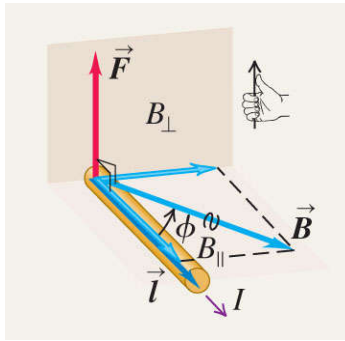
$$R = \frac{mv}{|q|B} \quad (27.11)$$



4. **Magnetic force on a conductor:** A straight segment of a conductor carrying current  $I$  in a uniform magnetic field  $\vec{B}$  experiences a force  $\vec{F}$  that is perpendicular to both  $\vec{B}$  and the vector  $\vec{l}$ , which points in the direction of the current and has magnitude equal to the length of the segment. A similar relationship gives the force  $d\vec{F}$  on an infinitesimal current-carrying segment  $d\vec{l}$ . (See Examples 27.7 and 27.8.)

$$\vec{F} = I\vec{l} \times \vec{B} \quad (27.19)$$

$$d\vec{F} = I d\vec{l} \times \vec{B} \quad (27.20)$$

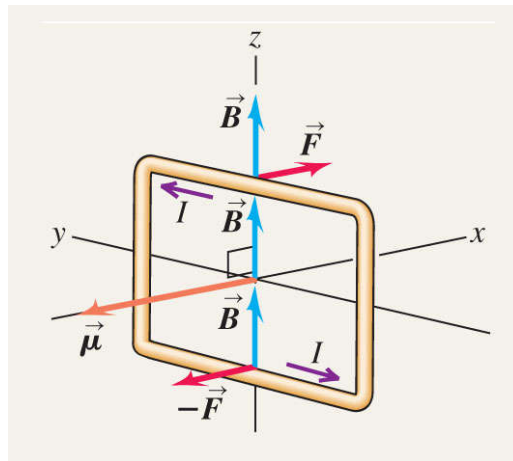


5. **Magnetic torque:** A current loop with area  $A$  and current  $I$  in a uniform magnetic field  $\vec{B}$  experiences no net magnetic force, but does experience a magnetic torque of magnitude  $\tau$ . The vector torque  $\vec{\tau}$  can be expressed in terms of the magnetic moment  $\vec{\mu} = I\vec{A}$  of the loop, as can the potential energy  $U$  of a magnetic moment in a magnetic field  $\vec{B}$ . The magnetic moment of a loop depends only on the current and the area; it is independent of the shape of the loop. (See Examples 27.9 and 27.10.)

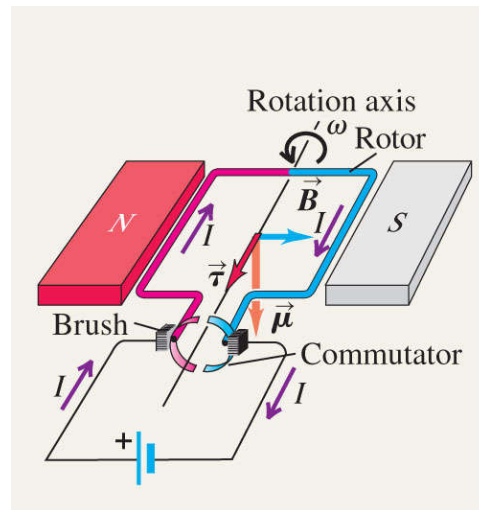
$$\tau = IBA \sin \phi \quad (27.23)$$

$$\vec{\tau} = \vec{\mu} \times \vec{B} \quad (27.26)$$

$$U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos \phi \quad (27.27)$$



6. **Electric motors:** In a dc motor a magnetic field exerts a torque on a current in the rotor. Motion of the rotor through the magnetic field causes an induced emf called a back emf. For a series motor, in which the rotor coil is in parallel with coils that produce the magnetic field, the terminal voltage is the sum of the back emf and the drop  $Ir$  across the internal resistance. (See Example 27.11.)



7. **The Hall effect:** The Hall effect is a potential difference perpendicular to the direction of current in a conductor, when the conductor is placed in a magnetic field. The Hall potential is determined by the requirement that the associated electric field must just balance the magnetic force on a moving charge. Hall-effect measurements can be used to determine the sign of charge carriers and their concentration  $n$ . (See Example 27.12.)

$$nq = \frac{-J_x B_y}{E_z} \quad (27.30)$$

