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A Model For Teaching Oil Shocks In A Small, Open, Oil-Exporting, And Developing Economy

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UNIVERSIDAD DEL NORTE
Vigilada Mineducación

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Companion proposed exercises to
Gómez-Ramírez and Quintero Otero (2026):

"A model for teaching oil shocks in a small, open, oil-exporting, and developing economy"

by Leopoldo Gómez-Ramírez*

September 2025

Of course, I have done my best to avoid mistakes or typos in this document.
Nevertheless, if you still find any, I'd appreciate you let me know

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Note: because they are set in continuous time, to solve some of the following exercises, students should know differential calculus.

1. In Gómez-Ramírez and Quintero Otero (2026) it's stated that, from the fact the nominal exchange rate (NER; E) is an inverse function of the price of oil (p_o) and of the nominal interest-rate (i), firms' price decisions (P), the real exchange rate definition (RER) —i.e., the amount of domestic goods that buy 1 foreign good: $\epsilon = \frac{EP^f}{P}$, where P^f denotes the foreign good price in foreign currency— and some conditions related to the elasticities of E with respect to i and p_o, it follows that ϵ is an inverse function of i and of p_o as well. Furthermore, the condition under which it is the case depends on whether something akin to Colombia's Fuel Prices Stabilization Fund (FPSF) operates or not. This exercise makes you over all this in detail. Therefore:

   i. Find the condition, in terms of the elasticities of W and E with respect to i, under which ϵ is an inverse function of i.

   ii. If nothing akin to the FPSF operates (so that increases/reductions in p_o do make the oil input in domestic currency more expensive/cheaper, thus exerting a force which makes Y more expensive/cheaper), find the condition, in terms of the elasticities of W and E with respect to p_o, under which ϵ is an inverse function of p_o.

   iii. If something like the FPSF operates (so that increases/reductions in p_o do not increase/reduce the cost of the oil input in domestic currency, thus for sure they make Y cheaper/more expensive, because they also make the imported input in domestic currency cheaper/more expensive), find the condition, in terms of the elasticities of W and E with respect to p_o, under which ϵ is an inverse function of p_o;

Answers:

i. First note that firm's price decisions are given by:

$$P = (1+m)(W + p^f_{IM}E + p_oE) \tag{1}$$

As we know, E is a function of i. But don't overlook that W is a function of i as well; because, as the supply-side analysis implies, it is a positive function of the employment gap $(N - N_e)$, which is a positive function of output gap $N - N_e$, which is an inverse function of i.[1] Thus, from Equation (1), it follows that

$$\frac{\partial P}{\partial i} = (1+m)\left(\frac{\partial W}{\partial i} + \left(p^f_{IM} + p_o\right)\frac{\partial E}{\partial i}\right) \tag{2}$$

For its part, from the definition of the RER, it follows that:

$$\begin{aligned}\frac{\partial \epsilon}{\partial i} &= \frac{\frac{\partial E}{\partial i}P^fP - \frac{\partial P}{\partial i}EP^f}{P^2} \\ &= \frac{P^f}{P^2}\left(\frac{\partial E}{\partial i}P - \frac{\partial P}{\partial i}E\right) \end{aligned} \tag{3}$$

[1]For brevity, to refer to $\frac{\partial W}{\partial i} = \frac{\partial W}{\partial (N-N_e)}\frac{\partial (N-N_e)}{\partial (Y-Y_e)}\frac{\partial (Y-Y_e)}{\partial i}$ and $\frac{\partial W}{\partial p_o} = \frac{\partial W}{\partial (N-N_e)}\frac{\partial (N-N_e)}{\partial (Y-Y_e)}\frac{\partial (Y-Y_e)}{\partial p_o}$, in what follows I'll just write down $\frac{\partial W}{\partial i}$ and $\frac{\partial W}{\partial p_o}$, respectively.

Thus, plugging (2) into (3) it follows that:

$$\begin{aligned}\frac{\partial \epsilon}{\partial i} &= \frac{P^f}{P^2}\left(\frac{\partial E}{\partial i}P - \left((1+m)\left(\frac{\partial W}{\partial i} + \left(p^f_{IM}+p_o\right)\frac{\partial E}{\partial i}\right)\right)E\right)\\ &= \frac{P^f}{P^2}\left(\frac{\partial E}{\partial i}\left(P-(1+m)(p^f_{IM}+p_o)E\right) - (1+m)\frac{\partial W}{\partial i}E\right)\end{aligned}$$

However, given that $P = (1+m)(W + (p^f_{IM}+p_o)E)$, then:

$$\begin{aligned}\frac{\partial \epsilon}{\partial i} &= \frac{P^f}{P^2}(1+m)\left(\frac{\partial E}{\partial i}W - \frac{\partial W}{\partial i}E\right)\\ &< 0 \text{ if } \tfrac{\partial E}{\partial i}W - \tfrac{\partial W}{\partial i}E < 0\end{aligned}$$

But, given that $\frac{\partial E}{\partial i} < 0$ and $\frac{\partial W}{\partial i} < 0$, then $\frac{\partial E}{\partial i}W - \frac{\partial W}{\partial i}E < 0$ holds if:

$$\left|\frac{\partial W}{\partial i}\right|\frac{i}{W} < \left|\frac{\partial E}{\partial i}\right|\frac{i}{E} \tag{4}$$

that is, if the elasticity of the wage with respect to the interest rate is, in absolute value, smaller than the elasticity of the exchange rate with respect to the interest rate is, in absolute value. (Thus, in Gómez-Ramírez and Quintero Otero, 2026, it is implictly assumed that condition 4 holds).

ii. As we know, E is a function of p_o. But don't overlook that W is a function of p_o as well; because, as the supply-side analysis implies, it is a positive function of the employment gap $(N - N_e)$, which is a positive function of output gap $N - N_e$, which is a function of p_o. Then, if nothing akin to the FPSF operates, from

Equation (1), it follows that:

$$\frac{\partial P}{\partial p_o} = (1+m)\left(\frac{\partial W}{\partial p_o} + (p^f_{IM} + p_o)\frac{\partial E}{\partial p_o} + E\right) \tag{5}$$

For its part, from the definition of the RER, it follows that:

$$\begin{aligned}\frac{\partial \epsilon}{\partial p_o} &= \frac{\frac{\partial E}{\partial i} P^f P - \frac{\partial P}{\partial p_o} E P^f}{P^2} \\ &= \frac{P^f}{P^2}\left(\frac{\partial E}{\partial p_o} P - \frac{\partial P}{\partial p_o} E\right)\end{aligned} \tag{6}$$

Thus, plugging (5) into (6) it follows that:

$$\begin{aligned}\frac{\partial \epsilon}{\partial p_o} &= \frac{P^f}{P^2}\left(\frac{\partial E}{\partial p_o} P - (1+m)\left(\frac{\partial W}{\partial p_o} + (p^f_{IM} + p_o)\frac{\partial E}{\partial p_o} + E\right) E\right) \\ &= \frac{P^f}{P^2}\left(\frac{\partial E}{\partial p_o}\left(P - (1+m)(P^f_{IM} + p_o)E\right) - (1+m)\left(\frac{\partial W}{\partial p_o} + E\right) E\right)\end{aligned}$$

However, given that $P = (1+m)(W + (p^f_{IM} + p_o)E)$, then:

$$\begin{aligned}\frac{\partial \epsilon}{\partial p_o} &= \frac{P^f}{P^2}(1+m)\left(\frac{\partial E}{\partial p_o} W - \left(\frac{\partial W}{\partial p_o} + E\right) E\right) \\ &< 0 \text{ if } \tfrac{\partial E}{\partial p_o} W - \left(\tfrac{\partial W}{\partial p_o} + E\right) E = \tfrac{\partial E}{\partial p_o} W - \tfrac{\partial W}{\partial p_o} E - E^2 < 0\end{aligned}$$

But, given that $\frac{\partial E}{\partial p_o} < 0$, then $\frac{\partial E}{\partial p_o} W - \frac{\partial W}{\partial p_o} E - E^2 < 0$ holds if either:

$$0 \leq \tfrac{\partial W}{\partial p_o} \tag{7}$$

$$\text{or } \tfrac{\partial W}{\partial p_o} < 0 \text{ and } \left|\tfrac{\partial W}{\partial p_o}\right| \tfrac{p_o}{W} < \left|\tfrac{\partial E}{\partial p_o}\right| \tfrac{p_o}{E} + \tfrac{E p_o}{W} \tag{8}$$

Condition (7) is the case if the output gap is a positive function of p_o (or independent of it). Condition (8) is the case if (8.a) the output gap is an inverse function of p_o and (8.b) the elasticity of the wage with respect to the oil price (which, we know, is in turn ultimately determined by the sensitivity of the output gap to oil price changes) is, in absolute value, smaller than the elasticity of the exchange rate with respect to the oil price, in absolute value, plus the value of the oil price in domestic currency in terms of the nominal wage. (Thus, in Gómez-Ramírez and Quintero Otero, 2026, it is implicitly assumed that either condition 7 or condition 8 holds).

iii. From Equation (1), it follows that:

$$\frac{\partial P}{\partial p_o} = (1+m)\left(\frac{\partial W}{\partial p_o} + (p^f_{IM} + p_o)\frac{\partial E}{\partial p_o} + E\right)$$

However, if something like Colombia's FPSF operates, then, given its operation make that the effects of variations of p_o on the cost of the oil input in domestic currency are not felt by the firms, we can state that $E + p_o\frac{\partial E}{\partial p_o} = 0$. But, then:

$$\frac{\partial P}{\partial p_o} = (1+m)\left(\frac{\partial W}{\partial p_o} + p^f_{IM}\frac{\partial E}{\partial p_o}\right) \tag{9}$$

For its part, from the definition of the RER, it follows that:

$$\begin{aligned}\frac{\partial \epsilon}{\partial p_o} &= \frac{\frac{\partial E}{\partial i} P^f P - \frac{\partial P}{\partial p_o} E P^f}{P^2} \\ &= \frac{P^f}{P^2}\left(\frac{\partial E}{\partial p_o} P - \frac{\partial P}{\partial p_o} E\right)\end{aligned}$$

Thus, plugging (9) into the latter, it follows that:

$$\begin{aligned}\frac{\partial \epsilon}{\partial p_o} &= \frac{P^f}{P^2}\left(\frac{\partial E}{\partial p_o} P - (1+m)\left(\frac{\partial W}{\partial p_o} + p^f_{IM}\frac{\partial E}{\partial p_o}\right) E\right) \\ &= \frac{P^f}{P^2}\left(\frac{\partial E}{\partial p_o}\left(P - (1+m)P^f_{IM}\right) - (1+m)\frac{\partial W}{\partial p_o} E\right)\end{aligned}$$

However, given that $P = (1+m)(W + p^f_{IM}E + p_o E)$, then:

$$\begin{aligned}\frac{\partial \epsilon}{\partial p_o} &= \frac{P^f}{P^2}(1+m)\left(\frac{\partial E}{\partial p_o}(W + p_o E) - \frac{\partial W}{\partial p_o} E\right) \\ &< 0 \text{ if } \tfrac{\partial E}{\partial p_o}(W + p_o E) - \tfrac{\partial W}{\partial p_o} E < 0\end{aligned}$$

But, given that $\frac{\partial E}{\partial p_o} < 0$, then $\frac{\partial E}{\partial p_o}(W + p_o E) - \frac{\partial W}{\partial p_o} E < 0$ holds if either:

$$0 \leq \tfrac{\partial W}{\partial p_o} \tag{10}$$

$$\text{or } \tfrac{\partial W}{\partial p_o} < 0 \text{ and } \left|\tfrac{\partial W}{\partial p_o}\right| \tfrac{p_o}{W} < \left|\tfrac{\partial E}{\partial p_o}\right| \tfrac{p_o}{E} + \left|\tfrac{\partial E}{\partial p_o}\right| \tfrac{(p_o)^2}{W} \tag{11}$$

As above said, condition (10) is the case if the output gap is a positive function of p_o (or independent of it). Condition (11) is the case if (11.a) the output gap is an inverse function of p_o and (11.b) the elasticity of the wage with respect to the oil price (which, we know, is in turn ultimately determined by the sensitivity of the output gap to oil price changes) is, in absolute value, smaller than the elasticity of the exchange rate with respect to the oil price, in absolute value, plus the sensitivity of the exchange rate with respect to the oil price time the oil price squared in terms of the nominal wage. (Thus, in Gómez-Ramírez and Quintero Otero, 2026, it is implicitly assumed that either condition 10 or 11 holds) (See Footnote 13 as well).

2. In Gómez-Ramírez and Quintero Otero (2026) we emphasized that E is a function of i and p_o. But, given our modeled economy exhibits some capital mobility, it is function of other variables. Then, find the other variables of which E is a function of.

Answer:

Denoting the foreign interest-rate by i^f and expectations about the future E by E^e_{+1}, the Uncovered Interest Parity (UIP) condition states that:

$$1 + i = (1 + i^f)\frac{E^e_{+1}}{E} \tag{12}$$

Solving (12) for E, it follows that:[2]

$$E = \left(\frac{1 + i^f}{1 + i}\right) E^e, \tag{13}$$

[2] Hereafter, to save on notation, instead of E^e_{+1} I'll just write down E^e.

Now, although we don't assume that the UIP fully holds in our economy (we don't assume that Equation 13 perfectly pins down E), from the incomplete operation of the UIP we can conclude that E is a positive function of i^f and E^e, and an inverse function of i. In addition of it, in our model E is an inverse function of p_o. Summarizing, then, the exchange rate is a function of the following variables as it follows:

$$\begin{aligned} &E(i, i^f, E^e, p_o) \text{ such that} \\ &\frac{\partial E}{\partial i} < 0,\ \ 0 < \frac{\partial E}{\partial i^f},\ \ 0 < \frac{\partial E}{\partial E^e}\ \ \&\ \ \frac{\partial E}{\partial p_o} < 0 \end{aligned} \tag{14}$$

3. In this exercise, ignore the effects of p_o on firm's costs and prices (and workers real wages and bargaining power), that is, on supply-side's "equilibrium output" concept, and focus only on its effects on spending decisions (aggregate demand). Furthermore, assume the Central Bank (CB) does not alter i (this is, thus, a totally "short-run" analysis). So, slightly different from Gómez-Ramírez and Quintero Otero (2026) specification, in which investment expenditures (I) are a positive function of aggregate demand (Y) and an inverse function of the interest-rate (i), suppose now that they are an inverse function of the NER (E), i.e.:

$$I\left(Y, i, E(p_o)\right) \text{ such that } \tfrac{\partial I}{\partial E} < 0 \tag{15}$$

The rationale for this assumption is that, in a developing economy, investment projects might also be inversely related to the NER, because an appreciated/depreciated currency (thus, appreciated/depreciated Y good), by making it cheaper/more expensive to acquire the imported inputs needed to carry investment projects, might boost/depress them. Then, discuss the effect of p_o on equilibrium aggregate demand.

Answer:

From Equation (15), it follows that

$$0 < \frac{\partial I}{\partial p_o} = \frac{\partial I}{\partial E}\frac{\partial E}{\partial p_o}$$

that is, it follows that investment expenditures are a positive function of p_o. From it, it follows that it is more likely —although we can't conclude it for sure— that equilibrium aggregate demand (Y) is a positive function of p_o; i.e., it's more likely that an increase/reduction in p_o boosts/depresses Y. This is the case because, in addition to its positive effects on C (via the reduction in T) and G, now p_o also positively affects I; therefore, it is more likely that the sum of those positive effects is greater than the inverse effect it exerts on the trade balance (recalling also that in Gómez-Ramírez and Quintero Otero, 2026, it is assumed that the Marshall-Lerner condition holds).

4. The spending decisions (aggregate demand) Equation presented in Gómez-Ramírez

and Quintero Otero (2026) (Equation 1 in it) is for the medium-run (i.e., P is not fixed). In this exercise you have to:

i. Find its short-run counterpart.

ii. According to your answer to item (i) (and the other assumptions of Gómez-Ramírez and Quintero Otero, 2026, model) find the other variables of which short-run equilibrium aggregate demand is function of.

Answers:

i. At first glance similarly-looking to its' medium-run counterpart, in the short run aggregate demand is given by:

$$Y = C + I + G + X - \epsilon M \tag{16}$$

with C, I, and G exactly the same as in Gómez-Ramírez and Quintero Otero (2026):

$$C(Y, T, i) \text{ such that } 0 < \tfrac{\partial C}{\partial Y},\ \tfrac{\partial C}{\partial T} < 0\ \&\ \tfrac{\partial C}{\partial i} < 0 \tag{17}$$

$$T(p_o) \text{ such that } \tfrac{\partial T}{\partial p_o} < 0 \tag{18}$$

$$I(Y, i) \text{ such that } 0 < \tfrac{\partial I}{\partial Y}\ \&\ \tfrac{\partial Y}{\partial i} < 0 \tag{19}$$

$$G(p_o) \text{ such that } 0 < \tfrac{\partial G}{\partial p_o} \tag{20}$$

However, given in the short-run both P and P^f are fixed, exports and imports are given by:

$$X[Y^f, E] \text{ such that } 0 < \tfrac{\partial X}{\partial Y^f} \ \& \ 0 < \tfrac{\partial X}{\partial E} \tag{21}$$

$$M[Y, E] \text{ such that } 0 < \tfrac{\partial M}{\partial Y} \ \& \ \tfrac{\partial M}{\partial E} < 0 \tag{22}$$

that is, they are function of the nominal, not the real exchange rate.

Substituting Equations (14) and (17) to (22) into Equation (16), the short-run aggregate demand Equation is obtained:

$$\begin{aligned} Y = {} & C(Y, T(p_o), i) + I(Y, i) + G(p_o) + X(Y^f, E(i, i^f, E^e, p_o)) \\ & - E(i, i^f, E^e, p_o) \left(\frac{P^f}{P} \right) M(Y, E(i, i^f, E^e, p_o)) \end{aligned} \tag{23}$$

in which I have explicitly written $\epsilon = E(i, i^f, E^e, p_o) \left(\frac{P^f}{P} \right)$ to highlight that, in the short-run, $\frac{P^f}{P}$ is fixed

ii. Equations (14), (17) to (22) (and assuming that the Marshall-Lerner condition in nominal terms holds) imply that the solution of (23) for Y (that is, short-run's equilibrium aggregate demand; denoted Y^{sr}) is a function of the following variables as it follows:

$$\begin{aligned} & Y^{sr}(i, i^f, E^e, p_o, Y^f, p_o) \text{ such that} \\ & \frac{\partial Y^{sr}}{\partial i} < 0, \ 0 < \frac{\partial Y^{sr}}{\partial i^f}, \ 0 < \frac{\partial Y^{sr}}{\partial E^e}, \ 0 < \frac{\partial Y^{sr}}{\partial Y^f} \ \& \ \frac{\partial Y^{sr}}{\partial p_o} \lesseqgtr 0 \end{aligned} \tag{24}$$

5. In this exercise, ignore the effects of p_o on firm's costs and prices (and workers real wages and bargaining power), that is, on the supply-side's "equilibrium output"; and focus only on its effects on spending decisions (equilibrium aggregate demand). Furthermore, assume the CB does not alter i (this is, thus, a totally "short-run" analysis). Also, assume that the Marshall-Lerner condition holds. So, in Gómez-Ramírez and Quintero Otero (2026) it is assumed that i is given by the "horizontal MP" ($i_t = i_t^{MP}$) which is a way to model it's chosen at discretion by the inflation-targeting (IT) Central Bank (with the aim of keeping π constant and equal to its target $0 < \pi^T$). However, in several undergraduate Macroeconomics materials the positively sloped "LM curve" (i.e., an LM expressing a positive relationship between i and Y coming from money market's equilibrium) is still used. This exercise is about comparing Gómez-Ramírez and Quintero Otero (2026) analysis of the effects of shocks on medium-run equilibrium aggregate demand (Y) with the analysis that would follows if such positively sloped LM holds. Thus, assume equilibrium i is determined by the following LM (of course, together with the IS relationship, that is, $Y = Y[i]$ such that $\frac{\partial Y}{\partial i} < 0$):

$$i = i(Y) \text{ such that } 0 < \frac{\partial i}{\partial Y} \tag{25}$$

In a graph in which Y is in the horizontal axis and i in the vertical axis, Equation (25) is a curve with positive slope. Assume too that, originally, the economy was initially at some equilibrium configuration in which $i^{MP} = i^{MP,O}$ and $Y = Y^O$.

i. If, then, p_o permanently decreases, what would happen? (Denote $i^{MP,N}$ and Y^N

to the new equilibrium values). How is it different from the analysis of the model with an horizontal MP?

ii. If, then, p_o permanently increases, what would happen? (Again, denote $i^{MP,N}$ and Y^N to the new equilibrium values). How is it different from the analysis of the model with an horizontal MP?

iii. What is the takeaway of these comparisons?

Answers:

i. Note, first, that —the same as in the model with the horizontal MP— we can't establish if the cheaper oil would boost or depress Y (or even leave it overall unchanged, but I'll skip this case). That is, in a graph with Y in the horizontal axis and i in the vertical axis, we can establish if the reduction in p_o would shift the IS to the right or to the left (or even leave it overall unchanged, but we'll skip this case). Instead, we can only say that:

   a. The IS shifts to the right if the contractionary domestic effects (C and G are reduced) are weaker than the expansionary trade balance effect ($X - \epsilon M$ increases).

   b. The IS shifts the the left if the contractionary domestic effects are stronger than the expansionary trade balance effects.

   Therefore, the analysis has to be carried out for each case.

   a. If the IS shifts to the right, then $Y^O < Y^N$ and $i^{MP,O} < i^{MP,N}$. These results are different from the results that would follow with the horizontal MP in two important ways. First, with the latter $i^{MP,O} = i^{MP,N}$, because the CB did not change it (by assumption of the "short-run" framework of this exercise). Second, with the horizontal MP, the increase in Y is greater than with the positively sloped LM. It follows from the fact that, with the positively sloped LM, the increase in i^{MP} depresses to some extent C and I (they still increase, but not as much as if i^{MP} remains constant) and, thus, the increase in Y is not as great as it would be if i^{MP} didn't change.

b. If the IS shifts to the left, then $Y^N < Y^O$ and $i^{MP,N} < i^{MP,O}$. These results are different from the results that (in this same case) would follow with an horizontal MP in two important ways. First, with the latter $i^{MP,O} = i^{MP,N}$ (by assumption of the "short-run" framework of this exercise). Second, with the horizontal MP, the reduction in Y is greater than with the positively sloped LM. It follows from the fact that, with the positively sloped LM, the reduction in i^{MP} boosts to some extent C and I (they still are depressed, but not as much as if i^{MP} remains constant) and, thus, the reduction in Y is not as great as it would be if i^{MP} didn't change.

ii. Again, first note that —the same as in the model with the horizontal MP— we can't establish if the more expensive oil would boost or depress Y (or even leave it overall unchanged, but I'll skip this case). That is, in a graph with Y in the horizontal axis and i in the vertical axis, we can't establish if the increase in p_o

would shift the IS to the right or to the left (or even leave it overall unchanged, but we'll skip this case). Instead, we can only say that:

a. The IS shifts to the right if the expansionary domestic effects (C and G are boosted) are stronger than the contractionary trade balance effects ($X - \epsilon M$ is depressed).

b. The IS shifts to the left if the expansionary domestic effects are weaker than contractionary trade balance effects.

Therefore, again, the analysis has to be carried out for each case. However, in case (a) the analysis is qualitatively the same as that of case (a) of item (i), and in case (b) the analysis is qualitatively the same as that of case (b) of item (i). The difference lies in the source, so to speak, of the IS's right or left shift, not in its results.

iii. The key takeaway is that, if the LM has positive slope, oil shocks exert smaller effects on equilibrium aggregate demand (whether positive or negative) than if the MP is horizontal. In other words, the horizontal LM magnifies the size of the effects of oil shocks on Y.

6. In Gómez-Ramírez and Quintero Otero (2026) it is just mentioned that the Marshall-Lerner condition is assumed to hold. In this exercise, you will dig deeper into what this assumption entails. So, consider the following good Y's trade balance (in terms of

domestic goods Y) function:

$TB = X(Y^f, \epsilon) - \epsilon M(Y, \epsilon)$ such that (the same as in the Gómez-Ramírez and Quintero Otero, 2026

$$0 < \frac{\partial X}{\partial Y^f}, \; 0 < \frac{\partial X}{\partial \epsilon}, \; 0 < \frac{\partial M}{\partial Y} \; \& \; \frac{\partial M}{\partial \epsilon} < 0 \tag{26}$$

Assume that, initially, the economy is in trade balance, i.e., $X - \epsilon M = 0$. Then:

i. Find the condition under which $\frac{\partial TB}{\partial p_o} < 0$ in terms of the elasticities of X and M with respect to the ϵ (that is, find the condition under which TB is an inverse function of p_o).

ii. Explain it in words.

Answers:

i. From Equation (26), it follows that

$$\begin{aligned}\frac{\partial TB}{\partial p_o} &= \frac{\partial X}{\partial \epsilon}\frac{d\epsilon}{dp_o} - \left(\frac{d\epsilon}{dp_o}M + \epsilon\frac{\partial M}{\partial \epsilon}\frac{d\epsilon}{dp_o}\right) \\ &= \frac{d\epsilon}{dp_o}\left(\frac{\partial X}{\partial \epsilon} - M - \epsilon\frac{\partial M}{\partial \epsilon}\right) \\ &= M\frac{d\epsilon}{dp_o}\left(\frac{1}{M}\frac{\partial X}{\partial \epsilon} - 1 - \frac{\epsilon}{M}\frac{\partial M}{\partial \epsilon}\right) \\ &< 0 \text{ if } 0 < \tfrac{\partial X}{\partial \epsilon}\tfrac{1}{M} - 1 - \tfrac{\partial M}{\partial \epsilon}\tfrac{\epsilon}{M}\end{aligned} \tag{27}$$

But, given the economy started in trade balance, then $\frac{1}{M} = \frac{\epsilon}{X}$. Also, recall that

$\frac{\partial M}{\partial \epsilon} < 0$. Therefore, the condition under which $\frac{\partial TB}{\partial p_o} < 0$ holds is:

$$1 < \frac{\partial X}{\partial \epsilon}\frac{\epsilon}{X} + \left|\frac{\partial M}{\partial \epsilon}\frac{\epsilon}{M}\right|$$

ii. The trade balance of good Y is inverse function of the price of oil (or positive function of the real exchange rate) if the sum of the elasticities of exports and imports (the latter in absolute value) with respect to the real exchange rate is greater than 1.

7. In Gómez-Ramírez and Quintero Otero (2026) it is highlighted that medium-run equilibrium aggregate demand (Y) is an inverse function of the interest-rate (i) and an indeterminate function of the price of oil (p_o) (i.e., Y is either a positive, a negative or even independent function of P_o). In this exercise, find the multitude of other variables of which Y is a function of.

Answer:

Medium-run aggregate demand is given by:

$$Y = C + I + G + X - \epsilon M \tag{28}$$

in which C is given by (17) and (18), I is given by (19), G is given by (20), the exports and imports functions are given by

$$X = X[Y^f, \epsilon] \text{ such that } 0 < \frac{\partial X}{\partial Y^f} \ \& \ 0 < \frac{\partial X}{\partial \epsilon} \tag{29}$$

$$M = M[Y, \epsilon] \text{ such that } 0 < \frac{\partial M}{\partial Y} \ \& \ \frac{\partial M}{\partial \epsilon} < 0 \tag{30}$$

and ϵ is (recall), defined as $\epsilon = \frac{EP^f}{P}$; so that ϵ is a function of P^f, all the variables affecting E (Equation 14) and all the variables affecting P (Equation 1), that is,

$$\epsilon = \epsilon(P^f, m, W, P^f_{IM}, i, i^f, E^e, p_o) \tag{31}$$

Substituting Equations (31), (17) to (20), (29), and (30) into Equation (28), the medium-run aggregate demand Equation is obtained:

$$\begin{aligned} Y = {} & C(Y, T(p_o), i) + I(Y, i) + G(p_o) + X(Y^f, \epsilon(P^f, m, W, P^f_M, i, i^f, E^e, p_o)) \\ & - \epsilon(P^f, m, W, P^f_{IM}, i, i^f, E^e, p_o)M(Y, \epsilon(P^f, m, W, P^f_{IM}, i, i^f, E^e, p_o)) \end{aligned} \tag{32}$$

Therefore, medium-run equilibrium aggregate demand is a function of the following variables:

$$Y = Y(i, i^f, E^e, Y^f, p_o, P^f, m, W, P^f_{IM})$$

However, as Equation (5) in Gómez-Ramírez and Quintero Otero (2026) implies, the current nominal wage is also a function of the last-period inflation rate, the last-period share of unit costs accruing to labor, the sensitivity of W to the employment gap (parameter β in Gómez-Ramírez and Quintero Otero, 2026), and the employment gap itself $(N - N_e)$; and the latter involves the (supply-side's) equilibrium employment level (N_e) (about which you will go over later in this set of exercises). Thus, in Gómez-Ramírez and Quintero Otero (2026)'s model Y is affected by all those (supply-side's) variables as well.

8. Assume the representative firm producing Y is an optimizing agent, in the specific sense it maximizes profits. Also, assume it faces a downward sloping demand curve; so that it has some power over the output price it sets. Specifically, let such demand curve be given by the following (constant-elasticity) function:

$$Y = P^{-\eta} \tag{33}$$

in which Y denotes output, P denotes its price, and we assume that the parameter η is greater than 1 and constant . For it's part, assume that the firm's cost function is (the simple linear function):

$$C(Y) = Y(W + P^f_{IM}E + p_oE) \tag{34}$$

in which W denotes the nominal wage, $P^f_{IM}E$ the domestic currency's price of the imported-input (P^f_{IM} denotes its foreign currency's price), and $p_o E$ the domestic currency's price of oil (p_o denotes its foreign currency's price).[3] Then:

i. Find the price-elasticity of demand that the firm faces.

ii. Write down the firm's profit maximization problem and solve it. Make sure the second order condition for a maximum Y is satisfied (so it indeed maximizes, not minimizes, profits).

iii. Thus, find the firm's optimal price decision.

iv. Explain the relationship between the price-elasticity of demand which you found in item (i) and the "markup" $0 < m$. Furthermore, highlight the two extreme cases captured by such relationship and explain them.

Answers:

i. From Equation (33), it follows that the price-elasticity of demand that the firm faces is (constant and equal to):

$$\frac{\mathrm{d}Y}{\mathrm{d}P}\frac{P}{Y} = -\eta;$$

which in absolute value is η.

ii. Note first that, from Equation (33) it follows that the inverse demand curve the firm faces is $P(Y) = Y^{-1/\eta}$. Together with the fact the firm's cost curve is given

[3] Note, by the way, that cost function (34) implies that unit costs and marginal costs are always the same: $\frac{\partial Y}{\partial Y} = \frac{C}{Y} = W + (P^f_{IM} + p_o)E$.

by (34), then the firm's profit-maximization problem is:

$$\begin{aligned}\max_{Y} B &= P(Y)Y - C(Y)\\ &= Y^{-1/\eta}Y - Y(W + P_M^f E + p_o E)\\ &= Y^{(\eta\,-\,1)/\eta} - Y(W + P_M^f E + p_o E)\end{aligned}$$

The first order condition of it is:

$$\frac{\partial B}{\partial Y} = \left(\frac{\eta - 1}{\eta}\right) Y^{-1/\eta} - (W + P_M^f E + p_o E) = 0 \tag{35}$$

And the second order condition of it is:

$$\frac{\partial^2 B}{\partial Y^2} = -\frac{1}{\eta}\left(\frac{\eta - 1}{\eta}\right) Y^{-(\eta\,+\,1)/\eta} \tag{36}$$

The expression in the right-hand side of Equation (36) is negative if (a) $1 < \eta$, which we actually assumed, and (b) $0 < Y$. So, if the optimal output which follows from (35) (moving forward, Y^*) turns out to be positive, then we can be sure that, in fact, Y^* maximizes (not minimizes) profits. Now, solving (35) for Y, it follows that the profit maximizing output is:

$$Y^* = \left(\left(\frac{\eta - 1}{\eta}\right)\left(\frac{1}{W + P_M^f E + p_o E}\right)\right)^{\eta} \tag{37}$$

From Equation (37) it follows that $0 < Y^*$ as long as (a) $1 < \eta$, which we actually assumed, and (b) $0 < W + P_M^f E + p_o E$, which we (obviously) assume. Therefore, Y^* given by Equation (37) in fact maximizes firm's profits.

iii. Evaluating the inverse demand function that the firm faces (recall, $P(Y) = Y^{-1/\eta}$) at the optimal Y^* given by 37, it is obtained that the optimal price (moving forward, P^*) the firm should set (at any time-period, t) is:

$$P_t^* = \left(\frac{\eta}{\eta - 1}\right)\left(W_t + P_{M,t}^f E_t + p_{o,t} E_t\right) = \left(1 + \left(\frac{1}{\eta - 1}\right)\right)(W_t + P_{M,t}^f E_t + p_{o,t} E_t)$$

So, if we define

$$m = \frac{1}{\eta - 1}, \tag{38}$$

it follows that:

$$P_t^* = (1 + m)(W_t + P_{M,t}^f E_t + p_{o,t} E_t),$$

which is the firms' price decision function posited in Gómez-Ramírez and Quintero Otero (2026).

iv. From Equation (38) it follows that there's an inverse relationship between the price-elasticity of demand that the firm faces, η, and m, which is the markup over unit costs it can charge to its costumers; specifically, $\frac{dm}{d\eta} = -\frac{1}{(\eta-1)^2} < 0$. There are two extreme cases worth highlighting in that relationship:

a. If $\eta \to \infty$ then $m \to 0$.

Explanation: if the price-elasticity demand that the firm faces is infinitely large, then its power to charge consumers a price above unit costs vanishes. It could be interpreted as the perfectly competitive markets case.

b. If $\eta \to 1$ then $m \to \infty$.

Explanation: if the price-elasticity of demand that the firm faces is small in the sense it approaches 1 (from numbers greater than 1, because recall we assumed $1 < \eta$), then its power to charge consumers a price above unit costs is infinitely large. It could be interpreted as the monopoly case.

9. Assume the following production function (which is more general than the one in Gómez-Ramírez and Quintero Otero, 2026, namely $Y = \min(N, I_M, O)$, because in the latter $\lambda = 1$, $\gamma = 1$ & $\theta = 1$):

$$Y = \min(\lambda N, \gamma I_M, \theta O) \tag{39}$$

in which $0 < \lambda$ but $\lambda \not\to \infty$ (i.e., labor's productivity is positive but not infinitely large), $0 < \gamma$ but $\gamma \not\to \infty$ (i.e., imported input's productivity is positive but not infinitely large).[4] Keep all other assumptions of Gómez-Ramírez and Quintero Otero (2026) model, including that something akin to Colombia's FPSF operates. Then:

i. Obtain the Phillips curve (PC) which follows from Equation (39).

ii. Explain why the qualitative message of the PC you obtained in item (i) is not

[4] Related to θ the operation of the FPSF's obviates the need to impose some restriction. However, of course, if $0 = \theta$ is allowed, then there's no point in having it the production function; and if $\theta \to \infty$ then there's no point in having it in the firm's unit cost expression.

different to the one conveyed by Gómez-Ramírez and Quintero Otero (2026)'s. Specifically, verify that the result that Y_{eq} (and, thus, Y_e) is a positive function of p_o is not altered.

iii. Allow for the possibility that (*ceteris paribus*) $\lambda \approx 0$ (infinitely close to 0). Obtain the PC which would then ensue. Explain its key messages (which now are certainly different to the PC in Gómez-Ramírez and Quintero Otero, 2026).

iv. Allow for the possibility that (*ceteris paribus*) $\lambda \to \infty$ (infinitely large). Obtain the PC which would then ensue. Explain its key messages (which now are certainly different to the PC in Gómez-Ramírez and Quintero Otero, 2026).

v. Allow for the possibility that (*ceteris paribus*) $\gamma \approx 0$ (infinitely close to 0). Obtain the PC which would then ensue. Explain its key messages (which now are certainly different to the PC in Gómez-Ramírez and Quintero Otero, 2026).

vi. Allow for the possibility that(*ceteris paribus*) $\gamma \to \infty$ (infinitely large). Obtain the PC which would then ensue. Explain its key messages (which now are certainly different to the PC in Gómez-Ramírez and Quintero Otero, 2026).

Answers:

i. Note, first, that, from the production function (39), it follows that the domestic currency's unit costs of production are:

$$\frac{W}{\lambda} + \frac{P_M E}{\gamma} + \frac{p_o E}{\theta}$$

Then, the PC that follows is[5] $\pi_t = \pi_{t-1} + (\beta/\lambda)\,(Y_t - Y_{n,t})\,\sigma_{N,t-1} + \rho\sigma_{M,t-1} + \tau\sigma_{O,t-1}$, where $\sigma_{N,t-1} = \frac{\gamma\theta W_{t-1}}{\gamma\theta W_{t-1}+\lambda\theta P_{M,t-1}E_{t-1}+\lambda\gamma p_{O,t-1}E_{t-1}}$ denotes the last period share of unit production costs accruing to labor, $\sigma_{M,t-1} = \frac{\lambda\theta P_{M,t-1}E_{t-1}}{\gamma\theta W_{t-1}+\lambda\theta P_{M,t-1}E_{t-1}+\lambda\gamma p_{o,t-1}E_{t-1}}$ the last period share of unit production costs accruing to the imported input, and $\sigma_{O,t-1} = \frac{\lambda\gamma p_{o,t-1}E_{t-1}}{\gamma\theta W_{t-1}+\lambda\theta P_{M,t-1}E_{t-1}+\lambda\gamma p_{o,t-1}E_{t-1}}$ the last period share of unit production costs accruing to the oil input. Now, under the assumption that the FPSF operates, the PC is:

$$\pi_t = \pi_{t-1} + (\beta/\lambda)\,(Y_t - Y_{n,t})\,\sigma_{N,t-1} + \rho\sigma_{M,t-1} \tag{40}$$

Equation (40) is different from Gómez-Ramírez and Quintero Otero (2026)'s PC in that in it $\sigma_{N,t-1}$ and $\sigma_{M,t-1}$ include the technological parameters λ, γ, and θ (and $\frac{1}{\lambda}$ multiplies the output gap).

ii. It could be seen that (as long as $\gamma \not\to \infty$ and the FPSF operates) the presence of the technological parameters λ, γ, and θ in Equation (40) does not qualitatively alter the reasoning such that an increase (reduction) in p_o reduces (increases) firms' unit costs and, thus increases Y_{eq}.

iii. If $\lambda \approx 0$ then $\sigma_{N,t-1} \approx 1$ and $\sigma_{M,t-1} \approx 0$. Therefore, Equation (40) can be approximated with:

$$\pi_t = \pi_{t-1} + (\beta/\lambda)\,(Y_t - Y_{e,t}) \tag{41}$$

[5] Step involved here: start with $P_t = (1+m)(\frac{W}{\lambda} + \frac{P_M E}{\gamma} + \frac{p_o E}{\theta})$, then apply the definition $\pi_t = \frac{P_t - P_{t_1}}{P_{t-1}}$, then substitute the $\hat{W}_t$ equation into the latter, and, of course, do some algebra.

The PC given by (41) states that —in addition of its last-period value— inflation would be totally driven by changes labor market and firm's price decisions which are not influenced by the imported input price growth rate (in domestic currency). This result is intuitive, because assuming that $\lambda \approx 0$ boils down to assume that, because labor productivity is almost null, infinitely large amounts of labor are needed for production.

iv. If $\lambda \to \infty$ then $\sigma_{N,t-1} \approx 0$. Therefore, Equation (40) can be approximated with:

$$\pi_t = \pi_{t-1} + \rho\sigma_{M,t-1} \tag{42}$$

The PC given by (42) states that —in addition of its last-period value— inflation would be totally driven by the imported input price growth rate (in domestic currency). This result is intuitive,, because assuming that $\lambda \to \infty$ boils down to assume that, because labor productivity is infinitely large, almost no labor is needed for production.

v. If $\gamma \approx 0$ then $\sigma_{N,t-1} \approx 0$ and $\sigma_{M,t-1} \approx 1$. Therefore, Equation (40) can be approximated with:

$$\pi_t = \pi_{t-1} + \rho \tag{43}$$

The PC given by (43) states that —in addition of its last-period value— inflation would be totally driven by the imported input price growth rate (in domestic currency). This result is intuitive, because assuming that $\gamma \approx 0$ boils down to assume that, because the imported input productivity is almost null, infinitely large amounts of the imported input are needed for production.

vi. If $\gamma \to \infty$ then $\sigma_{M,t-1} \approx 0$. Therefore, Equation (40) can be approximated with:

$$\pi_t = \pi_{t-1} + (^{\beta}/\lambda)(Y_t - Y_{n,t})\sigma_{N,t-1} \tag{44}$$

The PC given by (44) states that —in addition of its last-period value— inflation would be totally driven by changes labor market and firm's price decisions which are not influenced by the imported input price growth rate (in domestic currency). This result is intuitive, because assuming that $\gamma \to \infty$ boils down to assume that, because imported input productivity is infinitely large, almost no imported input is needed for production.

10. In this exercise, ignore the effects of p_o on firm's costs and prices (and workers real wages and bargaining power), that is, on the supply-side's equilibrium output; and focus only on its effects on spending decisions (equilibrium aggregate demand). Furthermore, assume the CB does not alter i (this is, thus, a totally "short-run" analysis).

    i. What are the effects of a permanent increase in p_o on the components of aggregate demand and the reasons explaining them? Assume that the Marshall-Lerner condition holds.

    ii. When is it the case that an increase in p_o boosts equilibrium aggregate demand

(Y)? When is it the case that an increase in p_o depresses Y? When is it the case an increase in p_o leaves Y overall unchanged? In this third case, does it mean that the components of aggregate demand are unchanged? Explain.

iii. Recalling the assumption that the horizontal MP does not change (that is, $i = i^{MP}$ remains the same) graph your responses to item (ii).

Answers:

i. If p_o permanently increases, then:

   C increases, because T are reduced.

   G increases, directly.

   The quantity of X decreases, because the NER (E) and RER $\epsilon)$ fall (given the assumptions of Gómez-Ramírez and Quintero Otero (2026), recall the exercise above about this issue), i.e., domestic currency and goods become more expensive.

   The quantity M increases, because the NER (E) and RER $\epsilon)$ fall (given the assumptions of Gómez-Ramírez and Quintero Otero (2026), recall the exercise above about this issue), i.e., foreign currency and goods become cheaper.

   Furthermore, because of the appreciation, the value of imports in terms of domestic goods (ϵM) may decrease. However, because of the Marshall-Lerner condition assumption, this value-effect is weaker than the X and M volume-effects. Thus, we conclude that the trade balance $(X - \epsilon M)$ is reduced.

ii. If the sum of the C and G expansionary effects is greater than the trade balance $X - \epsilon M$ contractionary effect, then Y increases.

Conversely, if the former is smaller than the latter, then Y decreases.

Finally, if both effects are exactly of the same size (rather unlikely but still possible to happen) then Y remains overall the same. It does not mean that the components of aggregate demand remain unchanged, because the domestic C and G are boosted while the trade balance $X - \epsilon M$ is reduced.

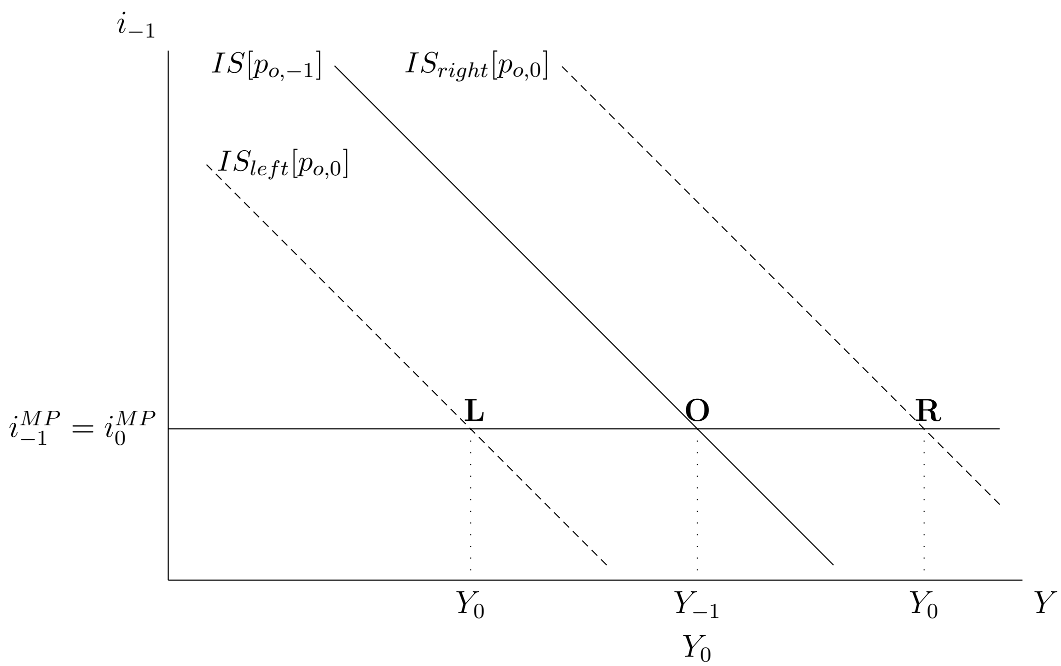
iii. Figure 1 graphically presents the three cases. It assumes the increase in p_o happens in $t = 0$ (that is, $p_{o,-1} < p_{o,-1}$) and the economy is, before the shock, in point **O**, with equilibrium aggregate demand equal to Y_{-1}.

If the expansionary domestic effects are weaker than the contractionary trade balance effect, the IS shifts to the left, and given the i^{MP} remains the same, the economy ends up at point **L**, with depressed equilibrium aggregate demand: $Y_0 < Y_{-1}$.

If the expansionary domestic effects are stronger than the contractionary trade balance effect, the IS shifts to the right, and given the i^{MP} remains the same, the economy ends up at point **R**, with boosted equilibrium aggregate demand: $Y_{-1} < Y_0$.

If the contractionary trade balance effect is of exactly the same size, in absolute value, than the expansionary domestic effects, the IS stays in the same position, and given the i^{MP} remains the same, the economy remains the very same point **O**, with equilibrium aggregate demand overall the same: $Y_{-1} = Y_0$ (but, of course, its composition changed).

Figure 1



11. In Gómez-Ramírez and Quintero Otero (2026) it is mentioned that, in a broader than Colombia's developing, oil-exporting economy context (in which p_o inversely affects E) but without the operation of a mechanism akin to FPSF's, the nature of the relationship between p_o and N_e is undetermined, i.e. the result that N_e is a positive function of p_o cannot be unequivocally established (thus, the results that Y_{eq} and Y_e are positive functions of p_o cannot be unequivocally established either). Solving this exercise, you will understand this issue in more detail. Thus, assume that nothing akin to the FPSF operates and the following production function (same as in Gómez-Ramírez and

Quintero Otero, 2026):[6]

$$Y = \min\left(N, I_M, O\right) \tag{45}$$

Then:

i. Obtain firms' domestic currency unit costs (UC) and explain why we cannot unequivocally establish the total effect of p_o on UC; which is why we cannot unequivocally establish the effect of p_o on N_e, Y_{eq}, and Y_e either.

ii. Find the condition under which we can, nevertheless, unequivocally establish that p_o inversely affects UC; which is the condition under which, even in the absence of something akin to FPSF operation, we can unequivocally establish that p_o positively affects N_e, Y_{eq}, and Y_e.

Answers:

i. From the production function (45) (and the other input prices' assumptions of the model) it logically follows $UC = W + p^f_{IM}E + p_oE$. But given that, $E(p_o)$, with $\frac{\partial E}{\partial p_o} < 0$, then, more explicitly:

$$UC = W + p^f_{IM}E(p_o) + p_oE(p_o) \text{ in which } \frac{\partial E}{\partial p_o} < 0 \tag{46}$$

[6]The exercise could be carried out with the more general production function, Equation (39) above.

From Equation (46), it follows that:

$$\frac{\partial UC}{\partial p_o} = p^f_{IM}\frac{\partial E}{\partial p_o} + E + p_o\frac{\partial E}{\partial p_o}$$
$$= \frac{\partial E}{\partial p_o}\left(p^f_{IM} + p_o\right) + E \tag{47}$$

Equation (47) show that, when p_o changes, it elicits opposing effects on UC. On the one hand (first term on the right-side summation of the second line of 47) it inversely affects UC, because it inversely affects the cost of the imported input and of the oil input in domestic currency (because it exerts a inverse effect on the exchange rate). On the other hand (second term on the right-side summation of the second line of 47) it positively affects UC, because it positively affects the foreign currency cost of the oil input. And it cannot be *a priori* established what of these two effects is stronger.

ii. From Equation (47) it follows that:

$$1 < \left|\frac{\partial E}{\partial p_o}\right|\frac{p_o}{E} + \left|\frac{\partial E}{\partial p_o}\right|\frac{p^f_{IM}}{E} \Rightarrow \frac{\partial UC}{\partial p_o} < 0$$

This condition expresses that, if, in absolute value, the sum of the elasticity of the exchange rate with respect to p_o and the (kind of elasticity) $\left|\frac{\partial E}{\partial p_o}\right|\frac{p^f_{IM}}{E}$ is greater than 1, then for sure UC are an inverse function of p_o.

12. This exercise is about verifying that the general expression for the inflation rate in

Gómez-Ramírez and Quintero Otero (2026), namely,

$$\pi_t = \omega_t \sigma_{N,t-1} + \rho_t \sigma_{M,t-1} + \tau_t \sigma_{O,t-1}, \tag{48}$$

was correctly obtained. Thus, take the Equation expressing firms' prices decisions, Equation (1) aforementioned but explicitly adding to it time subscripts:

$$P_t = (1+m)(W_t + p^f_{IM_t} E_t + p_{o,t} E_t) \tag{49}$$

Then, from Equation (49) and the discrete-time inflation rate definition ($\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$) obtain the expression for the latter in terms of wage growth (ω_t), domestic currency import input price growth (ρ), and domestic currency oil price growth (τ); i.e., obtain Equation (48).

Answer:

Note, first, that, from Equation (49), it follows that:

$$P_{t-1} = (1+m)\left(W_{t-1} + p^f_{IM,t-1} E_{t-1} + p_{o,t-1} E_{t-1}\right)$$

Then, applying the inflation rate definition:

$$\begin{aligned}
\pi_t &= \frac{P_t - P_{t-1}}{P_{t-1}} \\
&= \frac{(1+m)\left(W_t + p^f_{IM,t}E_t + p_{o,t}E_t\right) - (1+m)\left(W_{t-1} + p^f_{IM,t-1}E_{t-1} + p_{o,t-1}E_{t-1}\right)}{(1+m)\left(W_{t-1} + p^f_{IM,t-1}E_{t-1} + p_{o,t-1}E_{t-1}\right)} \\
&= \left(\frac{W_t - W_{t-1}}{W_{t-1} + p^f_{IM,t-1}E_{t-1} + p_{o,t-1}E_{t-1}}\right) + \left(\frac{p^f_{IM,t}E_t - p^f_{IM,t-1}E_{t-1}}{W_{t-1} + p^f_{IM,t-1}E_{t-1} + p_{o,t-1}E_{t-1}}\right) \\
&\quad + \left(\frac{p_{o,t}E_t - p_{o,t-1}E_{t-1}}{W_{t-1} + p^f_{IM,t-1}E_{t-1} + p_{o,t-1}E_{t-1}}\right) \\
&= \left(\frac{W_t - W_{t-1}}{W_{t-1}}\right)\left(\frac{W_{t-1}}{W_{t-1} + p^f_{IM,t-1}E_{t-1} + p_{o,t-1}E_{t-1}}\right) \\
&\quad + \left(\frac{p^f_{IM,t}E_t - p^f_{IM,t-1}E_{t-1}}{p^f_{IM,t-1}E_{t-1}}\right)\left(\frac{p^f_{IM,t-1}E_{t-1}}{W_{t-1} + p^f_{IM,t-1}E_{t-1} + p_{o,t-1}E_{t-1}}\right) \\
&\quad + \left(\frac{p_{o,t}E_t - p_{o,t-1}E_{t-1}}{p_{o,t-1}E_{t-1}}\right)\left(\frac{p_{o,t-1}E_{t-1}}{W_{t-1} + p^f_{IM,t-1}E_{t-1} + p_{o,t-1}E_{t-1}}\right) \\
&= \omega_t\sigma_{N,t-1} + \rho_t\sigma_{M,t-1} + +\tau_t\sigma_{O,t-1},
\end{aligned}$$

where $\omega_t = \frac{W_t - W_{t-1}}{W_{t-1}}$, $\rho_t = \frac{p^f_{IM,t}E_t - p^f_{IM,t-1}E_{t-1}}{p^f_{IM,t-1}E_{t-1}}$, $\tau_t = \frac{p_{o,t}E_t - p_{o,t-1}E_{t-1}}{p_{o,t-1}E_{t-1}}$,

$\sigma_{N,t-1} = \frac{W_{t-1}}{W_{t-1} + p^f_{IM,t-1}E_{t-1} + p_{o,t-1}E_{t-1}}$, $\sigma_{M,t-1} = \frac{p^f_{IM,t-1}E_{t-1}}{W_{t-1} + p^f_{IM,t-1}E_{t-1} + p_{o,t-1}E_{t-1}}$, and

$\sigma_{O,t-1} = \frac{p_{o,t-1}E_{t-1}}{W_{t-1} + p^f_{IM,t-1}E_{t-1} + p_{o,t-1}E_{t-1}}$; which is Equation (48).

13. In Gómez-Ramírez and Quintero Otero (2026) it is stated that adopting the assumption of inflation expectatios anchored to the Central Bank target inflation rate would significantly alter the analysis. This exercise is about this issue. Thus, consider the case in which private economic agents expect that, if needed, the CB will certainly intervene to achieve a constant and equal to its target inflation rate, no matter how

painful it could be. In this scenario, it could make sense to assume that inflation expectations are anchored to such CB's target, that is:

$$\pi_t^e = \pi^T \tag{50}$$

Keep all the other assumptions of the Gómez-Ramírez and Quintero Otero (2026) model. Then:

i. Obtain the Phillips curve which is the case if inflation expectations are given by Equation (50).

ii. Given the PC you obtained in item (i) (and assuming the economy was in medium-run-equilibrium before the shock) explain what would happen if there were a permanent reduction in p_o. For simplicity, assume that the latter does not depress equilibrium aggregate demand.

iii. How is your analysis of item (ii) different from that of Gómez-Ramírez and Quintero Otero (2026) (in which, recall, inflation expectations are totally adaptive: $\pi_t^e = \pi_{t-1}$)?

iv. Now keep all the other assumptions but, however, assume that (for some unknowns reason) there will be no CB's response to the shock, that is, $i_{-1}^{MP} = i_k^{MP}$ for $0 \leq k$ will be case. What would happen?

v. Based on your answer to item (iv), explain why it is important for the CB to foster that inflation expectations are anchored to its target.

Answers:

i. From Equation (50) it follows that wage growth is given by:[7]

$$\omega_t = \frac{\pi^T}{\sigma_N} + \beta(N_t - N_{e,t}) \tag{51}$$

Then (by writing 51 in terms of the output gap and substituting what it's obtained into the inflation rate expression), it follows that the PC is:

$$\pi_t = \pi^T + \beta\left(Y_t - Y_{eq,t}\right)\sigma_N + \rho_t\sigma_M$$

Or, defining $Y_{e,t} = Y_{eq,t} - \frac{\rho_t\sigma_M}{\beta\sigma_N}$ and $\alpha = \beta\sigma_N$:

$$\pi_t = \pi^T + \alpha\left(Y_t - Y_{e,t}\right) \tag{52}$$

Note that, from (52) it follows that:

$$\pi_t - \pi^T \lesseqgtr 0 \iff Y_t - Y_{e,t} \lesseqgtr 0$$

which shows that, in medium-run equilibrium (MRE), where the output gap is zero ($Y_t = Y_{e,t}$), the inflation rate aligns with the CB's target: $\pi_t = \pi^T$.

ii. ◦ Original MRE ($t = -1$)

[7] In what follows, the same as in Gómez-Ramírez and Quintero Otero (2026), for simplicity and without implying a qualitative change, we assume that σ_N and σ_M are constant.

The economy is originally ($t = -1$) at MRE, in which (a) $i_{-1} = i_{-1}^{MP}$, (b) $Y_{-1} = Y_{e,-1}$, and (c) $\pi_{-1} = \pi^T$.

- The shock's period ($t = 0$).

  Then, in $t = 0$, p_o experiences an exogenous one-time and permanent reduction: $p_{o,k} < p_{0,-1}$ for $0 \leq k$. Note, first, that the result that Y_e is a positive function of p_o is not changed by the fact that inflation expectations are anchored to the CB's target (Equation 50). Thus, to begin with, the reduction in p_o reduces Y_e, that is, $Y_e < Y_{e,-1}$. For its part, the effect of the reduction of p_o is undetermined. However, we assume that the cheaper oil price will not depress aggregate demand (i.e., $Y_{-1} \leq Y_0$).[8] Therefore, for sure a positive output gap would arise in the shock's period; say it's of size $0 < A$, that is, $0 < A = Y_0 - Y_{e,0}$. Consequently, the inflation rate will be above the CB's inflation target rate during the shock's period. Specifically, from Equation (52) it follows that $\pi^T < \pi_0 = \pi^T + \alpha(Y_0 - Y_{e,0}) = \pi^T + \alpha A$. But, then, as response, the CB will carry out a contractionary monetary policy, that is, in this period it will increase the interest rate: set $i_{-1}^{MP} < i_0^{MP}$. However, given its forward-looking and highly competent, the CB will increase it rightly forecasting (since this $t = 0$) (a) that next period's PC will be given that $\pi_1 = \pi^T + \alpha(Y_1 - Y_{e,1})$, because inflation expectations will remain anchored to its target, and (b) the exact amount in which, to achieve that $Y_1 - Y_{e,1} = 0$, i_0^{MP} has to be greater i_{-1}^{MP}.

[8]Of course, the following analysis does not hold if the cheaper oil boosts aggregate demand.

Don't overlook, then, the key difference with respect to the case in which inflation expectations are adaptive (and, thus, the PC of $t = 1$ would be given that $\pi_1 = \pi^T + \alpha A + \alpha(Y_1 - Y_{e,1})$). It's that, with inflation expectations anchored to the target, to bring π back to π^T, the CB does not have to create a negative gap in $t = 1$ (of size $-A$) and, after it, in $t = 2$ close the gap; instead, in a single step, in the very $t = 1$, it can just close the output gap.That is, the economy would be saved from an even more painful recession (although in the new MRE output will be reduced as well).

Summarizing, the economy ends up $t = 0$ with $Y_{e,0} < Y_{e,-1}$ and $Y_{-1} \leq Y_0$ such that (a) $0 < A = Y_0 - Y_{e,0}$, (b) $\pi^T < \pi_0$, and (c) $i_{-1}^{MP} < i_0^{MP}$.

- The new MRE ($t = 1$ and thereafter).

As consequence of the last-period's interest-rate increase, in $t = 1$ aggregate demand contracts, that is, $Y_1 < Y_0$, and equilibrium output expands, that is, $Y_{e,0} < Y_{e,1}$. Furthermore (because of the high CB's rationality and capabilities) these effects are such that they exactly close the output gap, that is, $Y_1 - Y_{e,1} = 0$. Consequently, the inflation rate will be $\pi_1 = \pi^T + \alpha(Y_1 - Y_{e,1}) = \pi^T + \alpha(0) = \pi^T$, that is, it'll align to the CB's target. Having achieved this outcome, the CB will no longer alter the interest rate: set $i_1^{MP} = i_0^{MP}$, which becomes the new stabilizing interest-rate. That is, because of its high rationality and capabilities, the CB rightly forecast, since $t = 1$, that, if it keeps the same interest-rate, next period's PC will be the same as this period's and, also, the output gap will remain zero.

Summarizing, the economy ends up $t = 1$ with $Y_1 < Y_0$ and $Y_{e,0} < Y_{e,1}$ such that (a) $Y_1 - Y_{e,1} = 0$, (b) $\pi_0 = \pi^T$, and (c) $i_1^{MP} = i_0^{MP}$.

The economy has reached a new MRE configuration. It's such that (a) inflation is constant and equal to the CB's target, i.e., $\pi^T = \pi_k = \pi_{k+1}$ for $1 \leq k$; (ii) actual and equilibrium output are permanently reduced, i.e., $Y_k = Y_{e,k} < Y_{e,-1} = Y_{-1}$ for $1 \leq k$; consequently, employment is permanently reduced as well; and (iii) the stabilizing interest rate is permanently higher, i.e., $i_{-1}^{MP} < i_k^{MP}$ for $0 \leq k$; which also implies that the domestic currency is permanently appreciated.

iii. The key difference it that, if inflation expectations are anchored to the CB's target, if a positive gap arises (in our case because equilibrium output decreased and aggregate demand was not reduced) so that the inflation rate is above its target, to lead the economy back to such target rate, the CB does not have to engage in the first-highly-contractionary-and-second-slightly-expansionary cycle; which it has to do when inflation expectations are adaptive, backward looking. Instead, it can just directly lead (depress) the economy towards the new and smaller equilibrium output.[9]

iv. If $i_{-1}^{MP} = i_k^{MP}$ for $0 \leq k$ is the case, then the following would happen.

[9]Symmetrically, if a negative output gap arises, so that the inflation rate decreases, to lead the economy back to the original equilibrium inflation rate (π^T) the CB does not have to engage in a first-highly-expansionary-and-secondly-slightly-contractionary cycle, as it has to do if inflation expectations are adaptive, backward looking. Instead, it can just directly lead (boost) the economy towards the new and greater equilibrium output.

- The shock's period ($t = 0$).

  The same as in item (ii) above except, of course, that $i_{-1}^{MP} = i_0^{MP}$ is the case; that is, the economy ends up $t = 0$ with $Y_{e,0} < Y_{e,-1}$ and $Y_{-1} \leq Y_0$ such that (a) $0 < A = Y_0 - Y_{e,0}$, (b) $\pi^T < \pi^T + A = \pi_0$, and (c) $i_{-1}^{MP} = i_0^{MP}$.

- Following to the period shock ($t = 1$) and thereafter.

  Given the last-period interest-rate remains unchanged, in $t = 1$ aggregate demand stays the same as well, that is, $Y_1 = Y_0$, and equilibrium output too, that is, $Y_{e,0} = Y_{e,1}$. Consequently, the same positive output gap remains, i.e., $0 < A = Y_1 - Y_{e,1}$. However, due to the fact that inflation expectations are anchored to the target, the inflation rate remains also the same (above the target but not above last-periods'); the following reasoning shows the latter:

$$\begin{aligned}\pi_1 &= \pi_1^e + \alpha(Y_1 - Y_{e,1}) \\ &= \pi^T + \alpha A \\ &= \pi_0\end{aligned}$$

  For its part, $i_1^{MP} = i_0^{MP}$ will again be the case. Summarizing, the economy ends up $t = 1$ with $Y_{e,1} = Y_{e,0}$ and $Y_1 = Y_0$ such that (a) $0 < A = Y_1 - Y_{e,1}$, (b) $\pi_0 = \pi^T + A = \pi_1$, and (c) $i_1^{MP} = i_0^{MP}$.

  But, because both $i_{-1}^{MP} = i_k^{MP}$ for $0 \leq k$ and, nevertheless, $\pi_k^e = \pi^T$ sticks for any k, then the just explained configuration will remain every other further period; that is, the economy will remain with $Y_{e,k} = Y_{e,0} < Y_{e,-1}$ for $1 \leq k$,

$Y_{-1} \leq Y_0 = Y_k$ for $1 \leq k$, such that (a) $0 < A = Y_k - Y_{e,k}$ for $0 \leq k$, (b) $\pi_k = \pi^T + A$ for $0 \leq k$, and (c) $i_{-1}^{MP} = i_k^{MP}$ for $0 \leq k$.

v. As it could be seen in item (iv) answer, if $\pi_t^e = \pi^T$ and, at the same time the CB did not respond to the shock (which does put π above its target), then, nevertheless, $\pi_k = \pi^T + A$ (in which $0 < A = Y_0 - Y_{e,0}$) for $0 \leq k$ will remain the case; that is, in the absence of CB actual intervention, an ever increasing inflation rate will not happen after the shock. This might be a key reason for which it's important for the CB to promote that inflation expectations are anchored to its target.

14. In order to understand the variables determining equilibrium employment (N_e) (and, thus, the level of output corresponding to it; which, given our production function is, $Y_{eq} = N_e$) in this exercise you are asked to examine the model that the Wage-Setting (WS) and Price-Setting (PS) Equations form, which captures labor market and firms' prices decisions —and that you could find in either Blanchard (2017, chapter 7) or Carlin and Soskice (2006, 2015, chapter 2)— and then extend it to our developing economy framework.[10] Therefore:

i. Write down the WS equation of either (i.e., implied by either) Blanchard (2017,

[10] Recall that we stated that, according to Blanchard (2017, chapter 7) or Carlin and Soskice (2006, 2015, chapter 2), N_e, and, thus, Y_{eq}, are (a) inverse functions of firms' market power, (b) inverse functions of workers' bargaining power, and (in Carlin and Soskice (2006, 2015, Chapter 2)) (c) positive functions of labor productivity (measured with $0 < \lambda$). However, as you know from the Gómez-Ramírez and Quintero Otero (2026) model and will further understand by solving this exercise's items (vi)-(ix) items, in our developing economy context, N_e (and, thus Y_{eq}) is function of a myriad of other variables (including p_o and i).

chapter 7) or Carlin and Soskice (2006, 2015, chapter 2) and explain its key messages; in a graph with employment (N) in the horizontal axis and the real wage ($\frac{W}{P}$) in the vertical axis, you may also want to sketch its graph.

ii. Write down the PS equation of either (i.e., implied by either) Blanchard (2017, chapter 7) or Carlin and Soskice (2006, 2015, chapter 2) and explain its key messages ; in a graph with N in the horizontal axis and $\frac{W}{P}$ in the vertical axis, you may also want to sketch its graph).

iii. When are both firms and workers in a situation in which they have no incentive (or power) to change their decisions, (i.e., they are in equilibrium)?

iv. Now assume the economy begins at some initial N_e and, then, firms' market power increases/decreases (i.e., m increases/decreases). How will N_e be in the new equilibrium configuration (moving forward, N_e^{new}). Why? In a graph with N in the horizontal axis and $\frac{W}{P}$ in the vertical axis, you may also want to carry out the analysis graphically.

v. Assume again that the economy begins at some initial N_e and, then, workers bargaining power increases/decreases. How will N_e be in the new equilibrium configuration. Why? In a graph with N in the horizontal axis and $\frac{W}{P}$ in the vertical axis, you may also want to carry out the analysis graphically.

vi. Now obtain the PS equation which is the case in our developing economy context. Also, for simplicity (although the model could also be extended in this was), assume the WS remains the same as the above's; furthermore, assume that something like Colombia's FPSF operates.

Also, here and in the following items disregard the effects that the exogenous variables have on the nominal wage and keep it fixed, that is, examine what happens to the real wage that the firms pay for a given nominal wage (but see Footnote 13 to grasp the thorny issue involved here).

vii. Assume again the economy begins at some initial N_e and, then, the foreign currency's price of the imported input (P^f_{IM}) increases/decreases. Given the WS-PS model of item vi, how will N_e be in the new equilibrium configuration. Why? In a graph with N in the horizontal axis and $\frac{W}{P}$ in the vertical axis, you may also want to carry out the analysis graphically.

viii. Other than for (a) variations in the foreign currency's price of the imported input (item vi), (b) variations in p_o, and (c) variations in i (you examined case (a) in item vii, while cases (b) and (c) were fully explained in Gómez-Ramírez and Quintero Otero (2026)), for what other reasons does the domestic currency's price of the imported input ($P^f_{IM}E[\cdot]$) change?

ix. Assume again the economy begins at some initial N_e and, then, the domestic currency's price of the imported input (P^f_{IM}) increases/decreases for any of the reasons you found in item viii. Given the WS-PS model of item vi, how will N_e be in the new equilibrium configuration. Why? In a graph with N in the horizontal axis and $\frac{W}{P}$ in the vertical axis, you may also want to carry out the analysis graphically.

x. Summarize, then, of what variables are N_e and, thus, Y_{eq} functions of.

Answers:[11]

i. The WS Equation given in Blanchard (2017, chapter 7) is:

$$W = PF(u, z) \text{ such that } \frac{\partial F}{\partial u} < 0 \;\&\; 0 < \frac{\partial F}{\partial z} \tag{53}$$

in which P denotes the price level, u denotes the unemployment rate, and z denotes the "catch-all" variable which "stands for all other variables that may affect the outcome of wage-setting" (Blanchard, 2017, p. 146); which includes, of course, worker's bargaining power. Therefore, Equation (53) conveys three messages:

- First, nominal wages are a positive function of the price level (in fact, Blanchard first posits that $W = P^e F[u, z]$ so that nominal wages are positive function of the price level expectations, but later assumes that $P^e = P$).
- Second, nominal wages are an inverse function of actual unemployment (positive function of actual employment).

[11] These answers using very similar to Blanchard (2017)'s Equations. However, because they convey the same qualitative messages, you could also solve the exercise using akin to Carlin and Soskice (2006, 2015)'s Equations

- Third, all the other factors which positively affect nominal wages are captured with z.

Solving (53) for the real wage it follows that $\frac{W}{P} = F(u, z)$. Thus, in a graph with N in the horizontal axis and $\frac{W}{P}$ in the vertical axis, the WS is an upward (always) sloping curve (or a line, we don't know, because we don't have some specific functional form of the F function).

ii. The PS Equation given in Blanchard (2017, chapter 7) is:

$$P = (1 + m)W \tag{54}$$

in which $0 < m$ denotes firms' market power (power to charge prices above their unit costs). Therefore, Equation (54) conveys two messages:

- First, the prices set by firms are a positive function of the nominal wages they have to pay to workers.
- Second, the prices set by firms are a positive function of the market power they enjoy.

Solving (54) for the real wage, it follows that $\frac{W}{P} = \frac{1}{1+m}$. It shows that the real wage firms pay is inverse function of their market power. In a graph with N in the horizontal axis and $\frac{W}{P}$ in the vertical axis, the PS is an horizontal line at $\frac{W}{P} = \frac{1}{1+m}$.[12]

[12] In Carlin and Soskice (2006, 2015) $P = (1 + m)\left(\frac{W}{\lambda}\right)$ in which $0 < \lambda$ denotes labor productivity, which is why (eventually) N_e turns out to be a positive function of λ as well.

iii. Neither firms nor workers have incentive —or power— to alter their wage and price decisions when the real wage and the unemployment rate going on in the economy simultaneously satisfy the WS and the PS relationships. Formally, this is the case at the $\frac{W}{P}$ and u which solve the system of equations (53)-(54) (moving forward, $\frac{W}{P}^{eq}$ and u^{eq}). However, given we don't have an explicit functional form for the F function, all we can conclude is that (a) $\frac{W}{P}^{eq} = \frac{1}{1+m}$ (because it follows from the PS alone) and (b) u^{eq} (which follows from solving $F(u,z) = \frac{1}{1+m}$). Also, don't overlook that, once u^{eq} is obtained, then N_e can be obtained too, because the latter is the N occurring in equilibrium (specifically, $N_e = L(1 - u^{eq})$, where L denotes the complete labor force).

    In a graph with N in the horizontal axis and $\frac{W}{P}$ in the vertical axis, the equilibrium is at the point in which the positively sloped WS and the horizontal PS intersect.

iv. As it could be seen from Equation (54), if m increases/decreases then firms will set a higher/lower P. But, then, for a given W, firms will pay a lower/greater $\frac{W}{P}$. And, then, to achieve that workers accept this lower/greater real wage (particularly, in order to achieve that workers lack the power to demand a higher W if $\frac{W}{P}$ decreases, because that could in turn yield a wage-price spiral) employment has to decrease/increase. This is the intuition behind the result that, if m increases/decreases then N_e decreases/increases.

    In a graph with N in the horizontal axis and $\frac{W}{P}$ in the vertical axis:

    - If m increases/increases, then the horizontal PS shifts down/up, thereby intersecting the positively sloped WS at a smaller/greater N, that is, N_e decreases/increases.

v. As it could be seen from Equation (53), if z increases/decreases then workers have the power to obtain increased/reduced nominal wages. But, then, as it could be seen from Equation (54), firms will increase/decrease the price they charge; in other words, they pass on their increased/decreased costs to final prices. And, therefore, it might create a wage-price spiral. For that to not happen, employment has to decrease/increase. This is the intuition behind the results that (a) if z increases/decreases then N_e decreases/increases and, conversely, (b) if z decreases/increases then N_e increases/decreases.

   In a graph with N in the horizontal axis and $\frac{W}{P}$ in the vertical axis:

   - If z increases/decreases, then the positively sloped WS shifts left/right, thereby intersecting the horizontal PS at a smaller/greater N, that is, N_e decreases/decreases.

vi. The price expression of our model is given Equation (1), which is worth writing down here again but explicitly including in it Equation (14):

$$P = (1+m)(W + (p_{IM}^f + p_o)E(i, i^f, E^e, p_o))$$

   Solving the latter for the real wage it follows that $\frac{W}{P} = \frac{W}{(1+m)(W+(p_{IM}^f+p_o)E(i,i^f,E^e,p_o)}$. This expression shows that, for a given nominal wage, the real wage firms pay is an inverse function of their market power (m), an inverse function of the foreign currency's imported input price (P_{IM}^f), and and inverse function of the nominal exchange rate (E) (it also shows that the real wage firms pay is undetermined function of p_o; but, given we assume something akin to Colombia's FPSF operates, we ignore so). But, given the exchange rate is in turn an inverse function of the interest rate (i), a positive function of the foreign interest rate (i^f), a positive function of inflation expectations (E^e), and an inverse function of the oil price (p_o), it follows that, for a given nominal wage, the real wage firms pay is a positive function of (i), an inverse function of i^f, an inverse function of E^e, and a positive function of p_o (recall, we assumed something akin to the FPSF operates).[13] Note that, in the graph in which $\frac{W}{P}$ is in the vertical axis and N in the horizontal axis, this expression is still an horizontal line.

vii. As it could be seen from Equation (1), if p^f_{IM} increases/decreases then firms will set a higher/lower P. But, then, for a given W, firms will pay a lower/greater $\frac{W}{P}$. And, then, to achieve that workers accept this lower/greater real wage (par-

[13] In fact, things are more complicated if we also take into account that the nominal wage changes when E changes (which, of course, in turn changes if either i, i^f, E^e, or p_o does), because the latter affect equilibrium aggregate demand, which in turn affects employment, which in turn affects the nominal wage. In this more detailed but complicated analysis, we would have to find the more specific conditions under which the results summarized in this item would hold. However, let me mention that:

- Related to the key Gómez-Ramírez and Quintero Otero (2026) model result such that the real wage, $\frac{W(i)}{P(i)} = \frac{W(i)}{(1+m)\left(W(i)+(P^f_{IM}+p_o)E(i)\right)}$ (making explicit that W is a function of i as well), is a positive function of i, it could be verified that it holds under condition (4) above, which we assumed.
- Related to the key Gómez-Ramírez and Quintero Otero (2026) model result such that, assuming the FPSF operates, the real wage, $\frac{W(p_o)}{P(p_o)} = \frac{W(p_o)}{(1+m)\left(W(p_o)+(P^f_{IM}+p_o)E(p_o)\right)}$ (making explicit that W is a function of p_o as well), is a positive function of p_o, it could be verified that it holds under either (i) condition (10) above, or (ii) condition (11.a) above together with condition $\left|\frac{\partial W}{\partial p_o}\right| \frac{p_o}{W} < \left|\frac{\partial E}{\partial p_o}\right| \frac{p_o}{E} \left(\frac{P^f_{IM}}{P^f_{IM}+p_o}\right)$. Then, we assume that either such (i) or (ii) holds. Note, by the way, that the second condition in (ii) is more stringent than condition (11.b) above. Thus, by assuming the former we guarantee that the latter holds too.

ticularly, in order to achieve that workers lack the power to demand a higher W if $\frac{W}{P}$ decreases, because that could in turn yield a wage-price spiral) employment has to decrease/increase. This is the intuition behind the results that, if p^f_{IM} increases/decreases, then N_e decreases/increases.

In a graph with N in the horizontal axis and $\frac{W}{P}$ in the vertical axis:

- If p^f_{IM} increases/increases, then the horizontal PS shifts down/up, thereby intersecting the positively sloped WS at a smaller/greater N, that is, N_e decreases/increases.

viii. (In addition of p^f_{IM}, i, and p_o) The domestic currency's price of the imported input (namely, $p^f_{IM}E(i, i^f, E^e, p_o)$) changes (positively in either case) if either i^f and E^e changes.

ix. As it could be seen from Equation (1), if either i^f increases/decreases or E^e increases/decreases (so that the domestic currency's price of the imported input increases/increases), then firms will set a higher/lower P. But, then, for a given W, firms will pay a lower/greater $\frac{W}{P}$. And, then, to achieve that workers accept this lower/greater real wage (particularly, in order to achieve that workers lack the power to demand a higher W if $\frac{W}{P}$ decreases, because that could in turn yield a wage-price spiral) employment has to decrease/increase. This is the intuition behind the results that, if either i^f increases/decreases or E^e increases/decreases, then N_e decreases/increases.

In a graph with N in the horizontal axis and $\frac{W}{P}$ in the vertical axis:

- If i^f increases/increases, then the horizontal PS shifts down/up, thereby intersecting the positively sloped WS at a smaller/greater N, that is, N_e decreases/increases.
- If E^e increases/increases, then the horizontal PS shifts down/up, thereby intersecting the positively sloped WS at a smaller/greater N, that is, N_e decreases/increases.

x. Summarizing, then, N_e is function of the following variables as it follows:

$$N_e(m, z, p^f_{IM}, i, i^f, E^e, p_o) \text{ such that}$$
$$\frac{\partial N_e}{\partial m} < 0,\ \frac{\partial N_e}{\partial z} < 0,\ \frac{\partial N_e}{\partial p^f_{IM}} < 0,\ 0 < \frac{\partial N_e}{\partial i},\ \frac{\partial N_e}{\partial i^f} < 0,\ \frac{\partial N_e}{\partial E^e} < 0 \ \&\ 0 < \frac{\partial N_e}{\partial p_o} \quad (55)$$

And, given that the production function implies that $Y_{eq} = N_e$, then Y_e is likewise function of the following variables as it follows:

$$Y_e(m, z, p^f_{IM}, i, i^f, E^e, p_o) \text{ such that}$$
$$\frac{\partial Y_e}{\partial m} < 0,\ \frac{\partial Y_e}{\partial z} < 0,\ \frac{\partial Y_e}{\partial p^f_{IM}} < 0,\ 0 < \frac{\partial Y_e}{\partial i},\ \frac{\partial Y_e}{\partial i^f} < 0,\ \frac{\partial Y_e}{\partial E^e} < 0 \ \&\ 0 < \frac{\partial Y_e}{\partial p_o} \quad (56)$$

15. Explain the reasons underpinning the result that, if p_o increases, then (assuming that something akin to the FPSF operates) N_e increases.

If p_o increases then the domestic currency appreciates. In turn, it makes the domestic currency's imported input (needed for production) cheaper and (also assuming (i) something like FPSF operates, so that the oil input price remains the same, and (ii) disregarding the effect of p_o the nominal wage (but see Footnote 13), thus, firms' unit costs are reduced. Consequently, firms set lower prices.[14] Therefore, the real wage is increased. But, then, an increase in N_e follows. The intuition of the latter is the following. If employment stayed below the new, greater N_e then workers would reduce their nominal wages hand in hand with the reduction in firms' prices.[15] However, it in turn would imply that the inflation rate decreases (and may imply a downward wage-price spiral). For that to not to happen, employment must be increased up to the new and greater N_e.

16. If something like the FPSF did not operate but keeeping the other model assumptions, Gómez-Ramírez and Quintero Otero (2026)'s PC would be:

$$\pi_t = \pi_{t-1} + \beta\left(Y_t - Y_{n,t}\right)\sigma_N + \rho_t\sigma_M + \tau_t\sigma_O \tag{57}$$

Given that the model's production function implies a one-to-one relationship between output and employment and, furthermore, there's an inverse relationship between the

[14]It follows from the price decisions Equation (and the assumption that something like FPSF operates): $P = (1+m)(W + (p^f_{IM} + p_o)E)$. But, of course, you may doubt firms actually reduce their prices when their costs are reduced (instead of pocketing such reduction as more profits). And there's no much I could tell you about it.

[15]Again, it follows from the wage setting specification: $W = P\cdot F(u,z)$. But, of course, you may doubt workers actually reduce their wages if the economy's prices are reduced (instead of pocketing such reduction as more purchasing power). And there's no much I could tell you about it.

employment level (N) and the unemployment rate (u) (recall, if L denotes the total labor force, then $N = L(1-u)$), then that PC can be rewritten as:

$$\pi_t = \pi_{t-1} - \delta\left(u_t - u_{e,t}\right) + v_t \tag{58}$$

Then:

i. Figure it out what δ denotes and what v_t denotes (don't overlook that v_t is a function of $p_{o,t}$).

For its part, the general Phillips curve of Mankiw (2007, chapter 13)'s textbook is:

$$\pi = \pi^e - \beta(u - u^n) + v$$

in which π^e denotes expected inflation, $u - u^n$ is a difference which Mankiw calls "cyclical unemployment",[16] and v is a "supply shock" term. In fact, explaining the addition of this last term to the "modern Phillips curve", (Mankiw, 2007, p. 387) says that "Credit for this addition goes to the OPEC" (Organization of Petroleum Exporting Countries) because, "In the 1970s, OPEC caused large increases in the world price of oil, which made economists more aware of the importance of shocks to aggregate supply". Therefore, in Mankiw (2007, Chapter 13)'s PC, v is a positive function of p_o.

Now, under the more specific assumption of adaptive inflation expectations, Mankiw

[16]When $v = 0$, it could be seen it captures that $\pi \lesseqgtr \pi^e \iff u^n \lesseqgtr u$.

(2007, chapter 13)'s PC becomes:

$$\pi = \pi_{-1} - \beta(u - u^n) + v \tag{59}$$

Therefore, Equations (58) and (59) look very similar. However:

ii. Explain why Equations (58) and (59) are fundamentally different in two aspects. Furthermore, explain the deeper reasons of the second difference (Note that, to answer this exercise, you'll likely have to read Mankiw, 2007, chapter 13).

Answers:

i. Substitute $N_t - N_{e,t}$ instead of $Y_t - Y_{e,t}$ (which follows from our production function) in Equation (57). Then use the facts that $N_t = L_t(1 - u_t)$ in general and $N_{e,t} = L_t(1 - u_{e,t})$ in particular. After some algebra, you'll obtain that:

$$\pi_t = \pi_{t-1} - \beta L \sigma_N \left(u_t - u_{e,t}\right) + \rho_t \sigma_M + \tau_t \sigma_O$$

You can see, then, that, defining $\delta = \beta L \sigma_N$ and $v_t = \rho_t \sigma_M + \tau_t \sigma_O$, the latter is Equation (58). Note, by the way, that $\frac{\partial v_t}{\partial p_{o,t}} \lesseqgtr 0$ (because $\frac{\partial \rho_t}{\partial p_{o,t}} < 0$ but $0 < \frac{d\tau_t}{dp_{o,t}}$).

ii. Equation (58) is different from (59) in two fundamental ways:

1. In Equation (58) u_n may be (although it's not for sure) inverse function of the price of oil (because N_e could be positive function of the price of oil) while in Equation (59) the price of oil does not exert some effect on u^n.[17]

[17]Interestingly, in Mankiw (2007, chapter 9)'s supply-side shocks' analysis, this scholar acknowledges that

2. In Equation (58) v may be (although it's not for sure) inverse function of p_o while in Equation (59) v is (fore sure) positive function of p_o. The deeper reason behind this difference could be summarized as it follows. Mankiw's analysis is written from the point of view of an economy which does not require imported inputs for production and, thus, does not need to examine the ways in which p_o could affect its exchange rate. Then, in Mankiw (2007, chapter 13)' PC, the v term captures all the things (in addition of a positive output gap) which positively affect the inflation rate; and, among them, the oil price prominently figures (because it only positively influences firms' costs).

17. In this exercise, you'll understand what would happen if, after an oil price shock, there were no CB's inflation-targeting intervention.[18] Assume then, that, at $t = -1$, the economy is in originally at MRE, in which (a) i_{-1} is the case, (b) $Y_{-1} = Y_{e,-1}$, and (c) $\pi_{-1} = \pi^T$. Then, in $t = 0$, p_o experiences an exogenous one-time and permanent reduction: $p_{o,k} < p_{o,-1}$ for $0 \leq k$. Assume too that, whatever its effects on the economy, the CB will not alter the i_{-1} prevailing before the shock ($t = -1$), that is, $i_{-1} = i_k$ for any $0 \leq k$ (so, no need to label it i^{MP}). Explain, then, what will happen?[19]

an adverse supply shock (which, from his developed economy's point of view, could come from a "raise in the world price of oil") "may also lower the natural level of output ... But we ignore that effect here" (Mankiw, 2007, p. 272). He also ignores such possibility in his Chapter 13's PC.

[18] For an analysis of the absence of the inflation-targeting policymaker in the dynamic context in which demand and/or supply are permanently growing/decreasing, you could see Davis and Gómez-Ramírez (2022, pp. 350-354).

[19] The same as in Gómez-Ramírez and Quintero Otero (2026), let me stress that what you will find by solving this exercise does not preclude the IT framework from being challenged on other grounds; particularly in developing economies contexts (Martins and Skott, 2021).

i Assuming the reduction in p_o does not depress aggregate demand (or it reduces it but not as much as it reduces equilibrium output), explain what would be the inflation rate at $t = 0$, $t = 1$, and $t = 2$. What's the path you could see will keep going on in the future?

ii. Assuming the reduction in p_o does depress aggregate demand and in a greater amount (in absolute value) than what it reduces equilibrium output, explain what would happen with the inflation rate at $t = 0$, $t = 1$, and $t = 2$. What's the path you could see will keep going on in the future?

iii. In this absence of stabilizing monetary policymaker scenario, what would have to be the effect of the reduction of p_o on aggregate demand for the inflation rate to remain constant?

Now return to the original ($t = -1$) MRE configuration (in which in which (a) i_{-1} is the case, (b) $Y_{-1} = Y_{e,-1}$, and (c) $\pi_{-1} = \pi^T$), and assume that, at $t = 0$, p_o experiences an exogenous one-time and permanent: $p_{o,-1} < p_{o,k}$ for $0 \leq k$. Assume again too that, whatever its effects on the economy, the CB will not alter the i_{-1} prevailing before the shock ($t = -1$), that is, $i_{-1} = i_k$ for any $0 \leq k$.

iv. Assuming the increase in p_o does not boost aggregate demand (or it boosts it but not as much as it boosts equilibrium output), explain what would happen with the inflation rate at $t = 0$, $t = 1$, and $t = 2$. What's the path you could see will keep going on in the future?

v. Assuming the increase in p_o does boost aggregate demand and in a greater amount than what it boosts equilibrium output, explain what would happen with the inflation rate at $t = 0$, $t = 1$, and $t = 2$. What's the path you could see will keep going on in the future?

vi. In this absence of stabilizing monetary policymaker scenario, what would have to be the effect of the oil price increase on aggregate demand for the inflation rate to remain constant?[20]

In items (i), (ii), (iv) and (v), find the exact mathematical values of the inflation rate each period; for that, it is convenient to call Y_e^N to the new equilibrium output and to denote each k-period's output gap with A, that is, $\alpha(Y_k - Y_e^N) = A \neq 0$ (as I'll do in the answers below).

Answers:

Recall, first, that inflation dynamics are given by the Phillips curve (Equation 8 in Gómez-Ramírez and Quintero Otero, 2026):

$$\pi_t = \pi_{t-1} + \alpha\left(Y_t - Y_{e,t}\right) \tag{60}$$

in which, recall, the effect $p_{o,t}$ on Y_t is undetermined but it exerts a positive effect on $Y_{e,t}$.

i. To begin with, the reduction in p_o reduces equilibrium output, i.e., $Y_e^N = Y_{e,0} <$

[20] Don't overlook the following. In item (iii) that the reduction in p_o does affect Y in exactly that way is, of course, very unlikely; likewise, in item (vi), that the increase in p_o does affect Ymr in exactly that way is a very unlikely. But, precisely because those exact effects are so unlikely is that they highlight the role of the stabilizing policy-maker.

$Y_{e,-1}$. Together with the fact the reduction in p_o does not depress aggregate demand (or it depresses it but not as much as it reduces Y_e) it follows that a positive output gap will arise at $t = 0$, i.e., $0 < Y_0 - Y_e^N$. Therefore, denoting $0 < A = \alpha(Y_0 - Y_e^N)$, from (60) it follows that:

$$\pi_0 = \pi_{-1} + \alpha \left(Y_0 - Y_e^N \right)$$

$$\pi_0 = \pi^T + A$$

Now, given the assumption that the CB will not change i_{-1}, it follows that, next period ($t = 1$, both $Y_1 = Y_0$ and $Y_e = Y_e^N$ again. But, then, the same positive output gap will arise at $t = 1$ too, i.e., $0 < A = \alpha(Y_1 - Y_e^N)$. And, then, from (60) it follows that:

$$\pi_1 = \pi_0 + \alpha \left(Y_1 - Y_e^N \right)$$

$$\pi_1 = \pi^T + A + A$$

$$\pi_1 = \pi^T + 2A$$

Now, given the assumption that the CB will not alter i_{-1}, it follows that, next period ($t = 2$), both $Y_2 = Y_1$ and $Y_e = Y_e^N$ again. But, then, the same positive output gap will arise at $t = 2$ too, i.e., $0 < A = \alpha(Y_2 - Y_e^N)$. And, then, from

(60) it follows that:

$$\pi_2 = \pi_1 + \alpha\left(Y_1 - Y_e^N\right)$$

$$\pi_2 = \pi^T + 2A + A$$

$$\pi_2 = \pi^T + 3A$$

Therefore, it could be deduced that as long as i_{-1} remains the case (i.e., there's no stabilizing monetary policymaker intervention), the inflation rate will keep increasing each period; specifically, each period it will grow in the amount $0 < A = \alpha(Y_0 - Y_e^N)$. Therefore, after k periods:

$$\pi_k = \pi^T + (k+1)A$$

And don't overlook that, despite the ever increasing inflation rate, actual output will remain at Y_0 every period, that is, it will not change after the initial shock.

ii. Now, the fact that the p_o reduction does depress aggregate demand and in a greater amount that what it reduces Y_e implies that a negative output gap will arise at $t = 0$, i.e., $Y_0 - Y_e^N = -A < 0$. Then, from (60) it follows that:

$$\pi_0 = \pi_{-1} + \alpha\left(Y_0 - Y_e^N\right)$$

$$\pi_0 = \pi^T - A$$

Now, given the assumption that the CB will not alter i_{-1}, it follows that, next period ($t = 1$), both $Y_1 = Y_0$ and $Y_e = Y_e^N$ again. But, then, the same negative output gap will arise at $t = 1$ too, i.e., $\alpha(Y_1 - Y_e^N) = -A < 0$. And, then, from (60) it follows that:

$$\pi_1 = \pi_0 + \alpha\left(Y_1 - Y_e^N\right)$$

$$\pi_1 = \pi^T - A - A$$

$$\pi_1 = \pi^T - 2A$$

Now, given the assumption that the CB will not alter i_{-1}, it follows that, next period ($t = 2$), both $Y_2 = Y_1$ and $Y_e = Y_e^N$ again. But, then, the same negative output gap will arise at $t = 2$ too, i.e., $\alpha(Y_2 - Y_e^N) = -A < 0$. And, then, from (60) it follows that:

$$\pi_2 = \pi_1 + \alpha\left(Y_2 - Y_e^N\right)$$

$$\pi_2 = \pi^T - 2A - A$$

$$\pi_2 = \pi^T - 3A$$

Therefore, it could be deduced that as long as i_{-1} remains the case (i.e., there's no stabilizing policymaker intervention), the inflation rate will keep decreasing each period; specifically, each period it will decrease in the amount $\alpha(Y_0 - Y_e^N) =$

$-A < 0$. Therefore, after k periods:

$$\pi_k = \pi^T - (k+1)A$$

Again, don't overlook that, despite the ever decreasing inflation rate, actual output will remain at Y_0 every period, that is, it will not change after the initial shock.

iii. If the reduction in p_o depressed aggregate demand in exactly the same amount (in absolute value) than what it depresses equilibrium output then, in $t = 0$, an output gap would not arise, i.e., $Y_0 - Y_e^N = 0$. Given the assumption i_{-1} will remain the case, it would then follow that $Y_k - Y_e^N = 0$ would be the case every other period $0 \leq k$. In this case, then, the inflation rate would remain constant (and equal to the prior ot the shock, CB's target: $\pi_{-1} = \pi^T$) even in the absence of the stabilizing monetary policymaker. Note that, finding this case in fact illuminates how unlikely it is that it occurs.

iv. To begin with, the increase in p_o increases equilibrium output, i.e., $Y_{e,-1} < Y_{e,0} = Y_e^N$. Together with the fact the reduction in p_o does not boost aggregate demand (or it boosts it but not as much as it boosts Y_e) it follows that a negative output gap will arise at $t = 0$, i.e., $Y_0 - Y_e^N < 0$. Therefore, denoting $\alpha(Y_0 - Y_e^N) =$

$-A < 0$, from (60) it follows that:

$$\pi_0 = \pi_{-1} - \alpha\left(Y_0 - Y_e^N\right)$$

$$\pi_0 = \pi^T - A$$

Now, given the assumption that the CB will not alter i_{-1}, it follows that, next period ($t = 1$), both $Y_1 = Y_0$ and $Y_e = Y_e^N$ again. But, then, the same negative output gap will arise at $t = 1$ too, i.e., $\alpha(Y_1 - Y_e^N) = -A < 0$. And, then, from (60) it follows that:

$$\pi_1 = \pi_0 + \alpha\left(Y_1 - Y_e^N\right)$$

$$\pi_1 = \pi^T - A - A$$

$$\pi_1 = \pi^T - 2A$$

Now, given the assumption that the CB will not alter i_{-1}, it follows that, next period ($t = 2$), both $Y_2 = Y_1$ and $Y_e = Y_e^N$ again. But, then, the same negative output gap will arise at $t = 2$ too, i.e., $\alpha(Y_2 - Y_e^N) = -A < 0$. And, then, from (60) it follows that:

$$\pi_2 = \pi_1 + \alpha\left(Y_2 - Y_e^N\right)$$

$$\pi_2 = \pi^T - 2A - A$$

$$\pi_2 = \pi^T - 3A$$

Therefore, it could be deduced that as long as i_{-1} remains the case (i.e., there's no stabilizing policymaker intervention), the inflation rate will keep decreasing each period; specifically, each period it will decrease in the amount $\alpha(Y_0 - Y_e^N) = -A < 0$. Therefore, after k periods:

$$\pi_k = \pi^T - (k+1)A$$

And don't overlook that, despite the ever decreasing inflation rate, actual output will remain at Y_0 every period, that is, it will not change after the initial shock.

v. Now the fact that the increase of p_o does boost aggregate demand and in a greater amount that what it boosts Y_e implies that a positive output gap will arise at $t = 0$, i.e., $0 < Y_0 - Y_e^N$. Therefore, denoting $0 < A = \alpha(Y_0 - Y_e^N)$, from (60) it follows that:

$$\pi_0 = \pi_{-1} + \alpha\left(Y_0 - Y_e^N\right)$$

$$\pi_0 = \pi^T + A$$

Now, given the assumption that the CB will not alter i_{-1}, it follows that, next period ($t = 1$), both $Y_1 = Y_0$ and $Y_e = Y_e^N$ again. But, then, the same positive output gap will arise at $t = 1$ too, i.e., $0 < A = \alpha(Y_1 - Y_e^N)$. And, then, from

(60) it follows that:

$$\pi_1 = \pi_0 + \alpha \left(Y_1 - Y_e^N\right)$$

$$\pi_1 = \pi^T + A + A$$

$$\pi_1 = \pi^T + 2A$$

Now, given the assumption that the CB will not alter i_{-1}, it follows that, next period ($t = 2$), both $Y_2 = Y_1$ and $Y_e = Y_e^N$ again. But, then, the same positive output gap will arise at $t = 2$ too, i.e., $0 < A = \alpha(Y_2 - Y_e^N)$. And, then, from (60) it follows that:

$$\pi_2 = \pi_1 + \alpha \left(Y_2 - Y_e^N\right)$$

$$\pi_2 = \pi^T + 2A + A$$

$$\pi_2 = \pi^T + 3A$$

Therefore, it could be deduced that as long as i_{-1} remains the case (i.e., there's no stabilizing policymaker intervention), the inflation rate will keep increasing each period; specifically, each period it will grow in the amount $0 < A = \alpha(Y_0 - Y_e^N)$. Therefore, after k periods:

$$\pi_k = \pi^T + (k + 1)A$$

And don't overlook that, despite the ever increasing inflation rate, actual output will remain at Y_0 every period, that is, it will not change after the initial shock.

vi. If the increase in p_o boosted aggregate demand in exactly the same amount than what it boosts equilibrium output then, in $t = 0$, an output gap would not arise, i.e., $Y_0 - Y_e^N = 0$. Given the assumption i_{-1} will remain the case, it would then follow that $Y_k - Y_e^N = 0$ would be the case every other period $0 \leq k$. In this case, then, the inflation rate would remain constant (and equal to the prior ot the shock, CB's target: $\pi_{-1} = \pi^T$) even in the absence of the stabilizing monetary policymaker. Note that, finding this case in fact illuminates how unlikely it is that it occurs.

18. In Gómez-Ramírez and Quintero Otero (2026) analysis of a permanent reduction in p_o, it is assumed it left aggregate demand overall unchanged (although, of course, its components change). However, as Gómez-Ramírez and Quintero Otero (2026) also stress, the effect of a variation in p_o on equilibrium aggregate demand is actually undetermined. In this exercise, thus, you'll carry out the analysis of a permanent reduction in p_o in all the remaining possible scenarios which arise according to the effect it exerts on Y. (Thus, this exercise is a detailed, period by period analysis of what was broadly examined in Gómez-Ramírez and Quintero Otero, 2026, subsecion 4.6). Therefore, assume that, originally ($t = -1$) the economy is at MRE, in which (a) $i_{-1} = i_{-1}^{MP}$, (b) $Y_{-1} = Y_{e,-1}$, and (c) $\pi_{-1} = \pi^T$. Then, at $t = 0$, p_o experiences an exogenous one-time and permanent increase: $p_{o,-1} < p_{o,k}$ for $0 \leq k$. According to

the Gómez-Ramírez and Quintero Otero (2026) model, explain what would happen in detail (period by period):

i. If the reduction in p_o boosts equilibrium aggregate demand;

ii. If the reduction in p_o reduces equilibrium aggregate demand but not as much as it reduces Y_e;

iii. If the reduction in p_o reduces equilibrium aggregate demand more than what it reduces Y_e;

For simplicity, in items (i)-(iii) assume that the CB is forward-looking highly competent so that it's able to fully bring back the economy towards an MRE configuration by $t = 2$.

iv. What is the other, not very likely but still possible case not covered by items (i)-(iii)? If it actually occurs, what would happen in the economy? How's stabilization different to that of items (i)-(iii)?

Answers:

i. ◦ The shock's period ($t = 0$)

To begin with, the reduction in p_o reduces Y_e, i.e., $Y_{e,0} < Y_{e,-1}$. And, given in this case it boosts aggregate demand (i.e., $Y_{-1} < Y_0$), then for sure a positive output gap arises in the shock's period; say it's of size $0 < \gamma A$, with $1 < \gamma$ to make it clear it's greater than that of Gómez-Ramírez and Quintero Otero (2026)' subsections 4.1–4.6 analysis, namely A (i.e., $0 < A < \gamma A = Y_0 - Y_{e,o}$).

This positive output gap will increase the inflation rate above the CB's target during the shock's period. Specifically, from Equation (60) it follows that:

$$\begin{aligned}\pi_0 &= \pi^T + \alpha(Y_0 - Y_{e,0}) \\ &= \pi^T + \alpha\gamma A \\ &> \pi^T\end{aligned}$$

But, then, as response, the CB will carry out contractionary monetary policy, which in this period means it'll increase the interest rate. i.e., set $i_{-1}^{MP} < i_0^{MP}$. However, given its forward-looking and highly competent, the CB will increase it rightly forecasting (since this $t = 0$) (a) that next period's PC will be given that $\pi_1 = \pi^T + \alpha\gamma A + \alpha(Y_1 - Y_{e,1})$, and (b) the exact amount in which i_0^{MP} has to be above i_{-1}^{MP} to achieve that $Y_1 - Y_{e,1} = -\gamma A < 0$.

Summarizing, the economy ends up $t = 0$ with $Y_{e,0} < Y_{e,-1}$ and $Y_{-1} < Y_0$ so that (a) $0 < \gamma A = Y_0 - Y_{e,0}$, (b) $\pi^T < \pi_0$, and (c) $i_{-1}^{MP} < i_0^{MP}$.

- Following to the shock's period ($t = 1$)

  As consequence of the last-period's interest-rate rise, in $t = 1$ aggregate demand decreases, i.e., $Y_1 < Y_0$, and equilibrium output increases, i.e., $Y_{e,0} < Y_{e,1}$. Furthermore (because of the high rationality and capabilities of the CB) these effects are such that they create a negative output gap which is equal, in absolute value, to the last-period's positive gap, i.e., $Y_1 - Y_{e,1} = -\gamma A < 0$.

Consequently, the inflation rate will be:

$$\begin{aligned}\pi_1 &= \pi^T + \alpha\gamma A + \alpha(Y_1 - Y_{e,1})\\ &= \pi^T + \alpha\gamma A - \alpha\gamma A\\ &= \pi^T,\end{aligned}$$

that is, the inflation rate aligns with the CB's target. Having achieved this outcome, then, in $t = 1$ the CB will reduce the interest rate, i.e., set $i_1^{MP} < i_0^{MP}$. However, given its forward-looking and highly competent, the CB will reduce it rightly forecasting (since $t = 1$) (a) that next period's PC will be given that $\pi_2 = \pi^T + \alpha(Y_2 - Y_{e,2})$, and (b) the exact amount in which i_1^{MP} has to be below i_0^{MP} to achieve that $Y_2 - Y_{e,2} = 0$.

Summarizing, the economy ends up $t = 1$ with $Y_1 < Y_0$ and $Y_{e,0} < Y_{e,1}$ such that (a) $Y_1 - Y_{e,1} = -\gamma A < 0$, (b) $\pi_1 = \pi^T$, and (c) $i_1^{MP} < i_0^{MP}$.

- The new MRE ($t = 2$ and thereafter)

  As consequence of last-period's interest-rate reduction, in $t = 2$ aggregate demand is boosted, i.e., $Y_1 < Y_2$, and equilibrium output reduced, i.e., $Y_{e,2} < Y_{e,1}$. Furthermore (because of the high rationality and capabilities of the CB) these effects are such that they exactly close the output gap, i.e., $Y_2 - Y_{e,2} = 0$.

Consequently, the inflation rate will be:

$$\begin{aligned}\pi_2 &= \pi^T + \alpha(Y_2 - Y_{e,2})\\ &= \pi^T + \alpha(0)\\ &= \pi^T,\end{aligned}$$

that is, it'll remain at the CB's target; furthermore, now it's constant as well: $\pi^T = \pi_2 = \pi_0$. Having achieved this outcome, the CB will no longer alter the interest rate, i.e., it'll set $i_2^{MP} = i_1^{MP}$, which becomes the new stabilizing interest-rate. More in detail, because of its high rationality and capabilities, the CB rightly forecast (since $t = 2$) that, if it keeps the same interest-rate, next period's PC will be the same as this period's and, also, the output gap will remain zero.

Summarizing, the economy ends up $t = 2$ with $Y_1 < Y_2$ and $Y_{e,2} < Y_{e,1}$ such that (a) $Y_2 - Y_{e,2} = 0$, (b) $\pi_2 = \pi^T$, and (c) $i_2^{MP} = i_1^{MP}$.

The economy has reached a new MRE configuration. It's such that (i) the inflation rate is constant and equal to the CB's target, i.e., $\pi^T = \pi_k = \pi_{k+1}$ for $1 \leq k$; (ii) the stabilizing interest rate is permanently higher, i.e., $i_{-1}^{MP} < i_k^{MP}$ for $1 \leq k$; which also implies that the currency is permanently appreciated; however, (iii) related to actual and equilibrium output, the three scenarios of Gómez-Ramírez and Quintero Otero (2026)'s Figure 6 arise:

iii.1. If the right shift of the IS is not strong enough so as to reach the IS that is labeled $IS[p_o^{low}]$ in Gómez-Ramírez and Quintero Otero (2026)'s Figure 6 middle panel, then actual and equilibrium output is permanently reduced, i.e., $Y_k = Y_{e,k} < Y_{e,-1} = Y_{-1}$ for $2 \leq k$ happens (thus, employment is permanently reduced as well). This is the scenario illustrated in Gómez-Ramírez and Quintero Otero (2026)'s Figure 6 upper panel.

iii.2. If the right shift of the IS is exactly as strong so as to reach the IS that is labeled $IS[p_o^{low}]$ in Gómez-Ramírez and Quintero Otero (2026)'s Figure 6 middle panel, then actual and equilibrium output is the same as in the original MRE's, i.e., $Y_k = Y_{e,k} = Y_{e,-1} = Y_{-1}$ for $2 \leq k$ happens (thus, employment is the same as well). This is the scenario illustrated in Gómez-Ramírez and Quintero Otero (2026)'s Figure 6 middle panel.

iii.3. If the right shift of the IS is so strong so that it goes beyond the IS that is labeled $IS[p_o^{low}]$ in Gómez-Ramírez and Quintero Otero (2026)'s Figure 6 middle panel, then actual and equilibrium output is permanently increases, i.e., $Y_{e,-1} = Y_{-1} < Y_k = Y_{e,k}$ for $2 \leq k$ happens (thus, employment is permanently increased as well). This is the scenario illustrated in Gómez-Ramírez and Quintero Otero (2026)'s Figure 6 lower panel.

ii. ◦ The shock's period ($t = 0$)

We know the reduction in p_o reduces Y_e, i.e., $Y_{e,0} < Y_{e,-1}$. And, given in this case it reduces aggregate demand (i.e., $Y_0 < Y_{-1}$) but not as much as it reduces equilibrium output, then for sure a positive output gap arises in the shock's period; say it's of size $0 < \delta A$, with $\delta < 1$ to make it clear it's smaller than that of Gómez-Ramírez and Quintero Otero (2026)'s subsections 4.1–4.5 analysis, namely A (i.e., $0 < \delta A = Y_0 - Y_{e,0} < A$). This positive output gap will posit above the target inflation rate during the shock's period. Specifically, from Equation (60) it follows that:

$$\begin{aligned}\pi_0 &= \pi^T + \alpha(Y_0 - Y_{e,0}) \\ &= \pi^T + \alpha\delta A \\ &> \pi^T\end{aligned}$$

But, then, as response, the CB will carry out contractionary monetary policy, which in this period means it'll increase the interest rate. i.e., set $i_{-1}^{MP} < i_0^{MP}$. However, given its forward-looking and highly competent, the CB will increase it rightly forecasting (since this $t = 0$) (a) that next period's PC will be given that $\pi_1 = \pi^T + \alpha\delta A + \alpha(Y_1 - Y_{e,1})$, and (b) the exact amount in which i_0^{MP} has to be above i_{-1}^{MP} to achieve that $Y_1 - Y_{e,1} = -\delta A < 0$.

Summarizing, the economy ends up $t = 0$ with $Y_{e,0} < Y_{e,-1}$ and $Y_0 < Y_{-1}$ so that (a) $0 < \delta A = Y_0 - Y_{e,0}$, (b) $\pi^T < \pi_0$, and (c) $i_{-1}^{MP} < i_0^{MP}$.

- Following to the shock's period ($t = 1$)

As consequence of the last-period's interest-rate rise, in $t = 1$ aggregate demand decreases, i.e., $Y_1 < Y_0$, and equilibrium output increases, i.e., $Y_{e,0} < Y_{e,1}$. Furthermore (because of the high rationality and capabilities of the CB) these effects are such that they create a negative output gap which is equal, in absolute value, to the last-period's positive gap, i.e., $Y_1 - Y_{e,1} = -\delta A < 0$. Consequently, the inflation rate will be:

$$\begin{aligned}\pi_1 &= \pi^T + \alpha\delta A + \alpha(Y_1 - Y_{e,1}) \\ &= \pi^T + \alpha\delta A - \alpha\delta A \\ &= \pi^T,\end{aligned}$$

that is, it aligns with the CB's target. Having achieved this outcome, then, in $t = 1$ the CB will reduce the interest rate, i.e., set $i_1^{MP} < i_0^{MP}$. However, given its forward-looking and highly competent, the CB will reduce it rightly forecasting (since $t = 1$) (a) that next period's PC will be given that $\pi_2 = \pi^T + \alpha(Y_2 - Y_{e,2})$, and (b) the exact amount in which i_1^{MP} has to be below i_0^{MP} to achieve that $Y_2 - Y_{e,2} = 0$.

Summarizing, the economy ends up $t = 1$ with $Y_1 < Y_0$ and $Y_{e,0} < Y_{e,1}$ such that (a) $Y_1 - Y_{e,1} = -\gamma A < 0$, (b) $\pi_1 = \pi^T$, and (c) $i_1^{MP} < i_0^{MP}$.

- The new MRE ($t = 2$ and thereafter)

  As consequence of last-period's interest-rate reduction, in $t = 2$ aggregate demand is boosted, i.e., $Y_1 < Y_2$, and equilibrium output is reduced, i.e., $Y_{e,2} < Y_{e,1}$. Furthermore (because of the high rationality and capabilities of the CB) these effects are such that they exactly close the output gap, i.e.,

$Y_2 - Y_{e,2} = 0$. Consequently, the inflation rate will be:

$$\pi_2 = \pi^T \alpha(Y_2 - Y_{e,2})$$
$$= \pi^T + \alpha(0)$$
$$= \pi^T,$$

that is, it'll remain at the CB's target; furthermore, now it's constant as well: $\pi^T = \pi_2 = \pi_0$. Having achieved this outcome, the CB will no longer alter the interest rate, i.e., it'll set $i_2^{MP} = i_1^{MP}$, which becomes the new stabilizing interest-rate. More in detail, because of its high rationality and capabilities, the CB rightly forecast (since $t = 2$) that, if it keeps the same interest-rate, next period's PC will be the same as this period's and, also, the output gap will remain zero.

Summarizing, the economy ends up $t = 2$ with $Y_1 < Y_2$ and $Y_{e,2} < Y_{e,1}$ such that (a) $Y_2 - Y_{e,2} = 0$, (b) $\pi_2 = \pi^T$, and (c) $i_2^{MP} = i_1^{MP}$.

The economy has reached a new MRE configuration. It's such that (i) inflation is constant and equal to the CB's target, i.e., $\pi^T = \pi_k = \pi_{k+1}$ for $1 \leq k$; (ii) actual and equilibrium output, is permanently reduced, i.e., $Y_k = Y_{e,k} < Y_{e,-1} = Y_{-1}$ for $2 \leq k$; consequently, employment is permanently reduced as well; and (iii) the stabilizing interest rate is permanently higher, i.e., $i_{-1}^{MP} < i_k^{MP}$ for $1 \leq k$; which also implies that the currency is permanently appreciated.

This is the scenario illustrated in Gómez-Ramírez and Quintero Otero (2026)'s Figure 5 upper panel.

iii. ○ The shock's period ($t = 0$)

We know the reduction in p_o reduces Y_e, i.e., $Y_{e,0} < Y_{e,-1}$. And, given in this case it reduces aggregate demand (i.e., $Y_0 < Y_{-1}$) and more than what it reduces equilibrium output, then for sure a negative output gap arises in the shock's period; say it's of size $0 < A$ (that is, $Y_0 - Y_{e,0} = -A < 0$). This negative output gap will posit below the target inflation rate during the shock's period. Specifically, from Equation (60) it follows that:

$$
\begin{aligned}
\pi_0 &= \pi^T + \alpha(Y_0 - Y_{e,0}) \\
&= \pi^T + \alpha(-A) = \pi^T - \alpha A \\
&< \pi^T
\end{aligned}
$$

But, then, as response, the CB will carry out expansionary monetary policy, which in this period means it'll reduce the interest rate. i.e., set $i_0^{MP} < i_{-1}^{MP}$. However, given its forward-looking and highly competent, the CB will decrease it rightly forecasting (since this $t = 0$) (a) that next period's PC will be given that $\pi_1 = \pi^T - \alpha A + \alpha(Y_1 - Y_{e,1})$, and (b) the exact amount in which i_0^{MP} has to be below i_{-1}^{MP} to achieve that $0 < A = Y_1 - Y_{e,1}$.

Summarizing, the economy ends up $t = 0$ with $Y_{e,-1} < Y_{e,0}$ and $Y_0 < Y_{-1}$ such that (a) $Y_0 - Y_{e,0} = -A < 0$, (b) $\pi_0 < \pi^T$, and (c) $i_0^{MP} < i_{-1}^{MP}$.

- Following to the shock's period ($t = 1$)

  As consequence of the last-period's interest-rate reduction, in $t = 1$ aggregate demand increases, i.e., $Y_0 < Y_1$, and equilibrium output decreases, i.e., $Y_{e,1} < Y_{e,0}$. Furthermore (because of the high rationality and capabilities of the CB) these effects are such that they create a positive output gap which is equal, in absolute value, to the last-period's negative gap, i.e., $0 < A = Y_1 - Y_{e,1} = 0$. Consequently, the inflation rate will be:

$$\begin{aligned}\pi_1 &= \pi^T - \alpha A + \alpha(Y_1 - Y_{e,1}) \\ &= \pi^T - \alpha A + \alpha A \\ &= \pi^T,\end{aligned}$$

  that is, it aligns with the CB's target. Having achieved this outcome, then, in $t = 1$ the CB will increase the interest rate, i.e., set $i_0^{MP} < i_1^{MP}$. However, given its forward-looking and highly competent, the CB will reduce it rightly forecasting (since $t = 1$) (a) that next period's PC will be given that $\pi_2 = \pi^T + \alpha(Y_2 - Y_{e,2})$, and (b) the exact amount in which i_1^{MP} has to be above i_0^{MP} to achieve that $Y_2 - Y_{e,2} = 0$.

  Summarizing, the economy ends up $t = 1$ with $Y_0 < Y_1$ and $Y_{e,1} < Y_{e,0}$ such that (a) $0 < A = Y_1 - Y_{e,1}$, (b) $\pi_1 = \pi^T$, and (c) $i_0^{MP} < i_1^{MP}$.

- The new MRE ($t = 2$ and thereafter)

  As consequence of last-period's interest-rate increase, in $t = 2$ aggregate demand is reduced, i.e., $Y_2 < Y_1$, and equilibrium output increases, i.e., $Y_{e,1} < Y_{e,2}$. Furthermore (because of the high rationality and capabilities of the CB) these effects are such that they exactly close the output gap, i.e., $Y_2 - Y_{e,2} = 0$. Consequently, the inflation rate will be:

$$\begin{aligned} \pi_2 &= \pi^T \alpha(Y_2 - Y_{e,2}) \\ &= \pi^T + \alpha(0) \\ &= \pi^T, \end{aligned}$$

that is, it'll remain at the CB's target; furthermore, now it's constant as well: $\pi^T = \pi_2 = \pi_0$. Having achieved this outcome, the CB will no longer alter the interest rate, i.e., it'll set $i_2^{MP} = i_1^{MP}$, which becomes the new stabilizing interest-rate. More in detail, because of its high rationality and capabilities, the CB rightly forecast (since $t = 2$) that, if it keeps the same interest-rate, next period's PC will be the same as this period's and, also, the output gap will remain zero.

Summarizing, the economy ends up $t = 2$ with $Y_2 < Y_1$ and $Y_{e,1} < Y_{e,2}$ such that (a) $Y_2 - Y_{e,2} = 0$, (b) $\pi_2 = \pi^T$, and (c) $i_2^{MP} = i_1^{MP}$.

The economy has reached a new MRE configuration. It's such that (i) the inflation rate is constant and equal to the CB's target, i.e., $\pi^T = \pi_k = \pi_{k+1}$ for $1 \leq k$; (ii) actual and equilibrium output are permanently reduced, i.e., $Y_k = Y_{e,k} < Y_{e,-1} = Y_{-1}$ for $2 \leq k$; consequently, employment is permanently reduced as well; and (iii) the stabilizing interest rate is permanently lower, i.e., $i_k^{MP} < i_{-1}^{MP}$ for $1 \leq k$; which also implies that the currency is permanently depreciated.

This is the scenario illustrated in Gómez-Ramírez and Quintero Otero (2026)'s Figure 5 lower panel.

iv. The other, not very likely but still possible case covered by neither items (i)-(iii) is that the reduction in p_o reduces aggregate demand in exactly the same amount in which it reduces equilibrium output. If it actually happens, the economy will experience the following stabilization process:

- The new MRE ($t = 0$ and thereafter)

  Recall, again, to begin with, that the reduction in p_o increases Y_e, i.e., $Y_{e,-1} < Y_{e,0}$. Now, if it reduces Y in the same amount (which, of course, implies that $Y_0 < Y_{-1}$) in which it reduces Y_e, then the output gap will remain 0 in the shock's period, i.e., $Y_0 - Y_{e,0} = 0$. This implies the inflation rate will remain at the CB's target. Specifically, from Equation (60) it follows that:

$$\begin{aligned}\pi_0 &= \pi^T + \alpha(Y_0 - Y_{e,0}) \\ &= \pi^T + \alpha(0) \\ &= \pi^T\end{aligned}$$

  But, then, the CB will will not change the interest-rate, i.e., it'll set $i_{-1}^{MP} =$

i_0^{MP}. More in detail, given its forward-looking and highly competent, the CB will rightly forecast (since this $t = 0$) (a) that next period's PC will be given that $\pi_1 = \pi^T + \alpha(Y_1 - Y_{e,1})$, and (b) keeping the same interest-rate it'll achieve $Y_1 - Y_{e,1} = 0$ again.

Summarizing, the economy ends up $t = 0$ with $Y_{e,0} < Y_{e,-1}$ and $Y_0 < Y_{-1}$ such that (a) $Y_0 - Y_{e,0} = 0$, (b) $\pi_0 = \pi^T$, and (c) $i_{-1}^{MP} = i_0^{MP}$.

But this already the new MRE configuration. It's such that (i) the inflation rate is constant and equal to the CB's target, i.e., $\pi^T = \pi_k = \pi_{k+1}$ for any k; (ii) actual and equilibrium output are permanently reduced, i.e., $Y_k = Y_{e,k} < Y_{e,-1} = Y_{-1}$ for $2 \leq k$; consequently, employment is permanently reduced as well; and (iii) the stabilizing interest rate is the same, i.e., $i_{-1}^{MP} = i_k^{MP}$ for any k. This is the scenario illustrated in the Gómez-Ramírez and Quintero Otero (2026)'s Figure 5 middle panel.

Comparison of item (iv) with items (i)-(iii) and Gómez-Ramírez and Quintero Otero (2026)'s subsections 4.1–4.5 analysis:

In this case, then, stabilization is less complicated than that of items (i) (in which actual output goes through a boom and then somewhat bust process), item (ii) (in which actual output goes through a bust, then further bust, and then somewhat boost process), item (iii) (in which actual output goes through a strong bust, then somewhat boom, then again bust process), and Gómez-Ramírez and Quintero Otero (2026)'s analysis (in which actual output goes through a staying the same, then bust, and then somewhat boost process). In this case, output just goes over a single bust process. Furthermore, stabilization does not require CB's intervention; it only has to rightly forecast than doing noting is it's right decision.

19. In Gómez-Ramírez and Quintero Otero (2026) and in the last exercise, we examined a permanent reduction in p_o. In this exercise, you'll examine in detail a permanent increase in p_o. Assume, thus, that, originally ($t = -1$) the economy is at MRE, in which (a) $i_{-1} = i_{-1}^{MP}$, (b) $Y_{-1} = Y_{e,-1}$, and (c) $\pi_{-1} = \pi^T$. Then, in $t = 0$, p_o experiences an exogenous one-time and permanent increase: $p_{o,-1} < p_{o,k}$ for $0 \leq k$. According to Gómez-Ramírez and Quintero Otero (2026) model, explain in detail (period by period) what would happen:

   i. If the increase in p_o does not boost aggregate demand (or boosts it but not as much as it boosts equilibrium output).

   ii. If the increase in p_o boosts aggregate demand more than what it boosts equilibrium output.

For simplicity, in items (i)-(ii) assume that the CB is forward-looking highly competent so that it's able to fully bring back the economy toward an MRE configuration in the very $t = 2$.

   iii. What is the other, not very likely but still possible case not covered by items (i) and (ii)? If it actually occurs, what would happen in the economy? How's stabilization different to that of item (i) and (ii)?

iv. Sketch one Figure (with one panel) that visually clarifies the many alternative final outcomes.[21] Explain.

Answers:

i. ∘ The shock's period ($t = 0$)

To begin with, the increase in p_o increases Y_e, i.e., $Y_{e,-1} < Y_{e,0}$. For its part, given that its effect on aggregate demand is that it does not boost it (or boost it but not as much as it boosts Y_e), then, for sure, a negative output gap arises in the shock's period; say it's of size $0 < A$ (that is, $Y_0 - Y_{e,0} = -A < 0$). This negative output gap will posit below the target inflation rate during the shock's period. Specifically, from Equation (60) it follows that:

$$\begin{aligned}\pi_0 &= \pi^T + \alpha(Y_0 - Y_{e,0}) \\ &= \pi^T + \alpha(-A) = \pi^T - \alpha A \\ &< \pi^T\end{aligned}$$

But, then, as response, the CB will carry out expansionary monetary policy, which in this period means it'll reduce the interest rate. i.e., set $i_0^{MP} < i_{-1}^{MP}$. However, given its forward-looking and highly competent, the CB will decrease it rightly forecasting (since this $t = 0$) (a) that next period's PC will be given that $\pi_1 = \pi^T - \alpha A + \alpha(Y_1 - Y_{e,1})$, and (b) the exact amount in

[21] You may also want to sketch analogous to Gómez-Ramírez and Quintero Otero (2026)'s Figures 5 and 6. But here I'm just asking for a single Figure (with one single panel) summarizing them all.

which i_0^{MP} has to be below i_{-1}^{MP} to achieve that $0 < A = Y_1 - Y_{e,1}$.

Summarizing, the economy ends up $t = 0$ with $Y_{e,-1} < Y_{e,0}$ and $Y_0 \leq Y_{-1}$ such that (a) $Y_0 - Y_{e,0} = -A < 0$ (or, even if $Y_{-1} < Y_0$ then, still, $Y_0 - Y_{e,0} < 0$), (b) $\pi_0 < \pi^T$, and (c) $i_0^{MP} < i_{-1}^{MP}$.

- Following to the shock's period ($t = 1$)

  As consequence of the last-period's interest-rate reduction, in $t = 1$ aggregate demand increases, i.e., $Y_0 < Y_1$, and equilibrium output decreases, i.e., $Y_{e,1} < Y_{e,0}$. Furthermore (because of the high rationality and capabilities of the CB) these effects are such that they create a positive output gap which is equal, in absolute value, to the last-period's negative gap, i.e., $0 < A = Y_1 - Y_{e,1} = 0$. Consequently, the inflation rate will be:

$$\begin{aligned}\pi_1 &= \pi^T - \alpha A + \alpha(Y_1 - Y_{e,1}) \\ &= \pi^T - \alpha A + \alpha A \\ &= \pi^T,\end{aligned}$$

  that is, it aligns with the CB's target. Having achieved this outcome, then, in $t = 1$ the CB will increase the interest rate, i.e., set $i_0^{MP} < i_1^{MP}$. However, given its forward-looking and highly competent, the CB will reduce it rightly forecasting (since $t = 1$) (a) that next period's PC will be given that $\pi_2 = \pi^T + \alpha(Y_2 - Y_{e,2})$, and (b) the exact amount in which i_1^{MP} has to be above i_0^{MP} to achieve that $Y_2 - Y_{e,2} = 0$.

Summarizing, the economy ends up $t = 1$ with $Y_0 < Y_1$ and $Y_{e,1} < Y_{e,0}$ such that (a) $0 < A = Y_1 - Y_{e,1}$, (b) $\pi_1 = \pi^T$, and (c) $i_0^{MP} < i_1^{MP}$.

- The new MRE ($t = 2$ and thereafter)

  As consequence of last-period's interest-rate increase, in $t = 2$ aggregate demand is reduced, i.e., $Y_2 < Y_1$, and equilibrium output increases, i.e., $Y_{e,1} < Y_{e,2}$. Furthermore (because of the high rationality and capabilities of the CB) these effects are such that they exactly close the output gap, i.e., $Y_2 - Y_{e,2} = 0$. Consequently, the inflation rate will be:

$$\begin{aligned}\pi_2 &= \pi^T \alpha(Y_2 - Y_{e,2}) \\ &= \pi^T + \alpha(0) \\ &= \pi^T,\end{aligned}$$

  that is, it'll remain at the CB's target; furthermore, now it's constant as well: $\pi^T = \pi_2 = \pi_0$. Having achieved this outcome, the CB will no longer alter the interest rate, i.e., it'll set $i_2^{MP} = i_1^{MP}$, which becomes the new stabilizing interest-rate. More in detail, because of its high rationality and capabilities, the CB rightly forecast (since $t = 2$) that, if it keeps the same interest-rate, next period's PC will be the same as this period's and, also, the output gap will remain zero.

  Summarizing, the economy ends up $t = 2$ with $Y_2 < Y_1$ and $Y_{e,1} < Y_{e,2}$ such that (a) $Y_2 - Y_{e,2} = 0$, (b) $\pi_2 = \pi^T$, and (c) $i_2^{MP} = i_1^{MP}$.

The economy has reached a new MRE configuration. It's such that (i) the inflation rate is constant and equal to the CB's target, i.e., $\pi^T = \pi_k = \pi_{k+1}$ for $1 \leq k$; (ii) actual and equilibrium output are permanently increased, i.e., $Y_{e,-1} = Y_{-1} = Y_k = Y_{e,k}$ for $2 \leq k$; consequently, employment is permanently reduced as well; and (iii) the stabilizing interest rate is permanently lower, i.e., $i_k^{MP} < i_{-1}^{MP}$ for $1 \leq k$; which also implies that the currency is permanently depreciated.

ii. ∘ The shock's period ($t = 0$)

We know the increase in p_o increases Y_e, i.e., $Y_{e,-1} < Y_{e,0}$. For its part, given that its effect on aggregate demand is that it boosts it more than what it boosts Y_e, then for sure a positive output gap arises in the shock's period; say it's of size $0 < A$ (that is, $0 < A = Y_0 - Y_{e,0}$). This positive output gap will posit above the target inflation rate during the shock's period. Specifically, from Equation (60) it follows that:

$$\begin{aligned}\pi_0 &= \pi^T + \alpha(Y_0 - Y_{e,0}) \\ &= \pi^T + \alpha A \\ &> \pi^T\end{aligned}$$

But, then, as response, the CB will carry out contractionary monetary policy, which in this period means it'll increase the interest rate. i.e., set $i_{-1}^{MP} < i_0^{MP}$. However, given its forward-looking and highly competent, the CB will decrease it rightly forecasting (since this $t = 0$) (a) that next period's PC will be given that $\pi_1 = \pi^T + \alpha A + \alpha(Y_1 - Y_{e,1})$, and (b) the exact amount in which i_0^{MP} has to be above i_{-1}^{MP} to achieve that $Y_1 - Y_{e,1} = -A < 0$.

Summarizing, the economy ends up $t = 0$ with $Y_{e,-1} < Y_{e,0}$ and $Y_{-1} < Y_0$ such that (a) $0 < A = Y_0 - Y_{e,0}$, (b) $\pi^T < \pi_0$, and (c) $i_{-1}^{MP} < i_0^{MP}$.

- Following to the shock's period ($t = 1$)

  As consequence of the last-period's interest-rate increase, in $t = 1$ aggregate demand is reduced, i.e., $Y_1 < Y_0$, and equilibrium output increases, i.e., $Y_{e,0} < Y_{e,1}$. Furthermore (because of the high rationality and capabilities of the CB) these effects are such that they create a negative output gap which is equal, in absolute value, to the last-period's positive gap, i.e., $Y_1 - Y_{e,1} = -A < 0$. Consequently, the inflation rate will be:

$$
\begin{aligned}
\pi_1 &= \pi^T + \alpha A + \alpha(Y_1 - Y_{e,1}) \\
&= \pi^T + \alpha A + \alpha(-A) \\
&= \pi^T,
\end{aligned}
$$

  that is, it aligns with the CB's target. Having achieved this outcome, then, in $t = 1$ the CB will reduce the interest rate, i.e., set $i_1^{MP} < i_0^{MP}$. However, given its forward-looking and highly competent, the CB will reduce it rightly forecasting (since $t = 1$) (a) that next period's PC will be given that $\pi_2 = \pi^T + \alpha(Y_2 - Y_{e,2})$, and (b) the exact amount in which i_1^{MP} has to be below

i_0^{MP} to achieve that $Y_2 - Y_{e,2} = 0$.

Summarizing, the economy ends up $t = 1$ with $Y_1 < Y_0$ and $Y_{e,0} < Y_{e,1}$ such that (a) $Y_1 - Y_{e,1} = -A < 0$, (b) $\pi_1 = \pi^T$, and (c) $i_1^{MP} < i_0^{MP}$.

- The new MRE ($t = 2$ and thereafter)

As consequence of last-period's interest-rate reduction, in $t = 2$ aggregate demand is increased, i.e., $Y_1 < Y_2$, and equilibrium output is reduced, i.e., $Y_{e,2} < Y_{e,1}$. Furthermore (because of the high rationality and capabilities of the CB) these effects are such that they exactly close the output gap, i.e., $Y_2 - Y_{e,2} = 0$. Consequently, the inflation rate will be:

$$\begin{aligned}\pi_2 &= \pi^T + \alpha(Y_2 - Y_{e,2}) \\ &= \pi^T + \alpha(0) \\ &= \pi^T,\end{aligned}$$

that is, it'll remain at the CB's target; furthermore, now it's constant as well: $\pi^T = \pi_2 = \pi_0$. Having achieved this outcome, the CB will no longer alter the interest rate, i.e., it'll set $i_2^{MP} = i_1^{MP}$, which becomes the new stabilizing interest-rate. More in detail, because of its high rationality and capabilities, the CB rightly forecasts that, since $t = 2$, that, if it keeps the same interest-rate, next period's PC will be the same as this period's and, also, the output gap will remain zero.

Summarizing, the economy ends up $t = 2$ with $Y_1 < Y_2$ and $Y_{e,2} < Y_{e,1}$ such that (a) $Y_2 - Y_{e,2} = 0$, (b) $\pi_2 = \pi^T$, and (c) $i_2^{MP} = i_1^{MP}$.

The economy has reached a new MRE configuration. It's such that (i) the inflation rate is constant and equal to the CB's target, i.e., $\pi^T = \pi_k = \pi_{k+1}$ for $1 \leq k$; (ii) actual and equilibrium output are permanently increased, i.e., $Y_{e,-1} = Y_{-1} = Y_k = Y_{e,k}$ for $2 \leq k$; consequently, employment is permanently increased as well; and (iii) the stabilizing interest rate is permanently greater, i.e., $i_{-1}^{MP} < i_k^{MP}$ for $1 \leq k$; which also implies that the currency is permanently appreciated.

iii. The other, not very likely but still possible case not covered in items (i) and (ii), is that the increase in p_o boosts aggregate demand in exactly the same amount than it boosts equilibrium output. If it actually happens, the economy will experience the following stabilization process:

- The new MRE ($t = 0$ and thereafter)

  Recall, again, to begin with, that the increase in p_o increases Y_e, i.e., $Y_{e,-1} < Y_{e,0}$. Now, if it increases Y in the same amount (which, of course, implies that $Y_{-1} < Y_0$) as it increases Y_e then the output gap will remain 0 in the shock's period, i.e., $Y_0 - Y_{e,o} = 0$. This implies the inflation rate will remains at the CB's target. Specifically, from Equation (60) it follows that:

$$\pi_0 = \pi^T + \alpha(Y_0 - Y_{e,0})$$
$$= \pi^T + \alpha(0)$$
$$= \pi^T$$

But, then, the CB will will not change the interest-rate i.e., it'll set $i_{-1}^{MP} = i_0^{MP}$. More in detail, given its forward-looking and highly competent, the CB will rightly forecast (since this $t = 0$) (a) that next period's PC will be given that $\pi_1 = \pi^T + \alpha(Y_1 - Y_{e,1})$, and (b) keeping the same interest-rate it'll achieve $Y_1 - Y_{e,1} = 0$ again.

Summarizing, the economy ends up $t = 0$ with $Y_{e,-1} < Y_{e,0}$ and $Y_{-1} < Y_0$ such that (a) $Y_0 - Y_{e,0} = 0$, (b) $\pi_0 = \pi^T$, and (c) $i_{-1}^{MP} = i_0^{MP}$.

But this already the new MRE configuration. It's such that (i) the inflation rate is constant and equal to the CB's target, i.e., $\pi^T = \pi_k = \pi_{k+1}$ for any k; (ii) actual and equilibrium output are permanently increased, i.e., $Y_{e,-1} = Y_{-1} < Y_k = Y_{e,k}$ for $2 \leq k$; consequently, employment is permanently increased as well; and (iii) the stabilizing interest rate is the same, i.e., $i_{-1}^{MP} = i_k^{MP}$ for any k.

Comparison of item (iii) with item (i) and (ii):

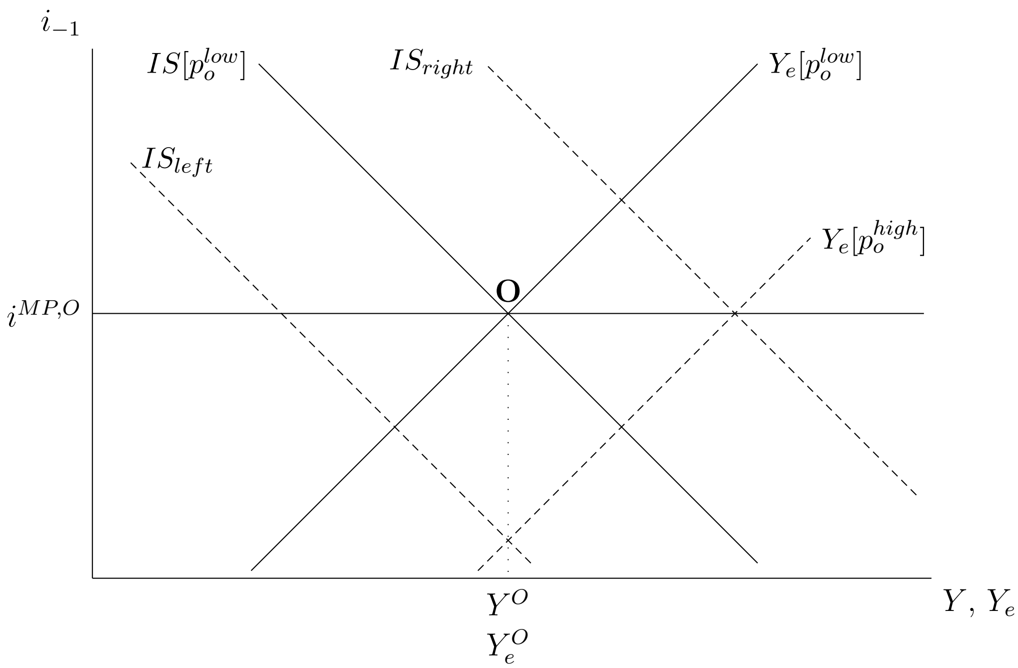
In this case, then, stabilization is less complicated than that of item (i) (in which actual output goes through a boom and then somewhat bust process) and that of item (ii) (in which actual output goes through a strong boom, then an strong bust, and finally an smaller boost process). In this case, output just goes over a single boom process. Furthermore, stabilization does not require CB's intervention; it only has to rightly forecast than doing noting is it's right decision.

iv. With both Y_e and Y in the horizontal axis and i in the vertical axis, Figure 2 of this document shows that the original MRE is in the intersection of the Y_e and IS curves prevailing before the reduction in p_o (which I label $Y_e[p_o^{low}]$ and $IS[p_o^{low}]$), point **O**; and that the new MRE configuration will lie in the intersection of the Y_e and IS curves ensuing after p_o increases. Now, we do know that the latter will shift the Y_e curve to the right; thus, I label the emerging new Y_e curve as $Y_e[p_o^{high}]$ and show it is to the right of $Y_e[p_o^{low}]$. Then, given such $Y_e[p_o^{high}]$ curve, the new MRE configuration depends on the direction and strength of the shift (if any) of the IS curve that the increase in p_o elicits. But, therefore, we can visualize three general scenarios:

   i. if the IS shifts to the right very strongly so that it lies to the right of the IS curve which I label IS$_{right}$ then in the new MRE output will be permanently increased (to the right of the original $Y^O = Y_e^O$) and the interest-rate will be permanently greater (above the original $i^{MP,O}$);

   ii. if the IS shifts to the right, stays overall the same, or shifts to the left but in any case falls within the range between the IS$_{right}$ and IS which we label IS$_{left}$, then, in the new MRE output will be permanently increased (to the

Figure 2: Increase in p_o: alternative scenarios



right of the original $Y^O = Y_e^O$) and the interest-rate will be permanently lower (below the original $i^{MP,O}$);

iii. if the IS shifts to the left very strongly so that it lies to the left of the IS$_{left}$ curve, then in the new MRE output will be permanently reduced (to the left of the original $Y^O = Y_e^O$) and the interest-rate will be permanently lower (below the original $i^{MP,O}$).

Note, by the way, that, after the increase in p_o, the only scenario we can preclude is that both output is permanently reduced and the interest-rate is permanently higher.

20. In Gómez-Ramírez and Quintero Otero (2026) analysis of a permanent reduction in p_o, CB's response is analyzed in terms of the nominal interest-rate (i, which is the interest-

rate the CB actually sets) and its just mentioned (Gómez-Ramírez and Quintero Otero, 2026, Foonote 7) that, if the effects of the real interest-rate (r) on effective output, Y, were considered (given it's r the relevant variable affecting C and I), then, as long as consumers and investors inflation expectations are also adaptive (and the CB rightly forecast so) no qualitative change would be obtained (which is why we chose not to unnecessarily complicate the analysis in that way). In this exercise you are going to verify it.

Therefore, with $\pi^e_{t+1} = \frac{P^e_{t+1} - P_t}{P_t}$, take the following exact relationship between the real and the nominal interest rates (explained in Blanchard, 2017, chapter 6 in detail):

$$1 + r_t = \frac{1 + i_t}{1 + \pi^e_{t+1}} \tag{61}$$

and assume that Y is a function of r (not only of i). Assume also that, originally ($t = -1$) the economy is at MRE, in which (a) $i_{-1} = i^{MP}_{-1}$, (b) $Y_{-1} = Y_{e,-1}$, and (c) $\pi_{-1} = \pi^T$. Then, in $t = 0$, p_o experiences an exogenous one-time and permanent reduction: $p_{o,k} < p_{o,-1}$ for $0 \leq k$.[22] For simplicity, assume that the CB is forward-looking highly competent so that it's able to fully bring back the economy towards an MRE configuration by $t = 2$.

Then, verify in detail, that is, period by period, that the analyses of Gómez-Ramírez and Quintero Otero (2026) and an exercise above, carried out in terms of i, are not qualitative altered; that is, verify that the relationships between i_{-1} and i_0, i_0 and i_1,

[22] I encourage you to also verify that no qualitative change would follow in the analysis of the case of an increase in p_o either

and i_1 and i_2 that are obtained in those analyses are not qualitative changed neither if:

i. the reduction in p_o boosts equilibrium aggregate demand or reduces it but not as much as it reduces Y_e; so that, in any case, a positive output gap arises in $t = 0$;

nor if:

ii. the reduction in p_o reduces equilibrium aggregate demand more than what it reduces Y_e, so that a negative output gap arises in $t = 0$;

Answers:

Note, first, that from Equation (61) it follows that:

$$r_t = \frac{1 + i_t}{1 + \pi^e_{t+1}} - 1$$

Given the assumption that consumers and investors have adaptive inflation expectations, $\pi^e_{t+1} = \pi_t$, the latter implies that the CB sets r_t as given by:

$$r_t = \frac{1 + i_t}{1 + \pi_t} - 1 \tag{62}$$

i. ◦ The shock's period ($t = 0$)

In this case, the reduction in p_o for sure creates a positive output gap in the shock's period; say it's of size $0 < A$ (that is, $0 < A = Y_0 - Y_{e,0}$). This positive output gap will put the inflation rate above the CB's target during the shock's period. Specifically, from Equation (60) it follows that:

$$
\begin{aligned}
\pi_0 &= \pi^T + \alpha(Y_0 - Y_{e,0}) \\
&= \pi^T + \alpha A
\end{aligned}
$$

Given the adaptive inflation expectations, it implies that $\pi_1^e = \pi_0 = \pi^T + \alpha A$. Then, as response, the CB will carry out contractionary monetary policy, which in this period means it'll increase the real interest rate. i.e., set $r_{-1}^{MP} < r_0^{MP}$. But, from Equation (62) it follows the CB sets the nominal interest-rate i_0^{MP} such that the following holds:

$$
\begin{aligned}
\frac{1 + i_{-1}^{MP}}{1 + \pi_0^e} - 1 &< \frac{1 + i_0^{MP}}{1 + \pi_1^e} - 1 \\
\frac{1 + i_{-1}^{MP}}{1 + \pi^T} &< \frac{1 + i_0^{MP}}{1 + \pi^T + \alpha A}
\end{aligned}
$$

which, after some algebra, boils down to:

$$
\frac{i_{-1}^{MP} + (1 + i_{-1}^{MP})\alpha A}{1 + \pi^T} < i_0^{MP}
$$

But note that:

$$
i_{-1}^{MP} < \frac{i_{-1}^{MP} + (1 + i_{-1}^{MP})\alpha A}{1 + \pi^T}
$$

Therefore, the qualitative result obtained before, namely, $i_{-1}^{MP} < i_0^{MP}$, is not altered.[23]

Given its forward-looking and highly competent, the CB will increase r_0 rightly forecasting (since this $t = 0$) (a) that next period's PC will be given that $\pi_1 = \pi^T + \alpha A + \alpha(Y_1 - Y_{e,1})$, and (b) the exact amount in which r_0^{MP} has to be above r_{-1}^{MP} to achieve that $Y_1 - Y_{e,1} = -A < 0$.

Summarizing, the economy ends up $t = 0$ with $Y_{e,0} < Y_{e,-1}$ and $Y_{-1} < Y_0$ so that (a) $0 < A = Y_0 - Y_{e,0}$, (b) $\pi^T < \pi_0$, and (c) $r_{-1} < r_0$; the latter in turn implies that $i_{-1}^{MP} < i_0^{MP}$, as verified.

- Following to the shock's period ($t = 1$)

As consequence of the last-period's interest-rate rise, in $t = 1$ aggregate demand decreases, i.e., $Y_1 < Y_0$, and equilibrium output increases, i.e., $Y_{e,0} < Y_{e,1}$. Furthermore (because of the high rationality and capabilities of the CB) these effects are such that they create a negative output gap which is equal, in absolute value, to the last-period's positive gap, i.e., $Y_1 - Y_{e,1} = -A < 0$. Consequently, the inflation rate will be:

$$\begin{aligned}
\pi_1 &= \pi^T + \alpha A + \alpha(Y_1 - Y_{e,1}) \\
&= \pi^T + \alpha A - \alpha A \\
&= \pi^T,
\end{aligned}$$

that is, it aligns with the CB's target. Given the adaptive inflation expec-

[23] Actually, it verifies that $i_{-1}^{MP} < i_0^{MP}$ is necessary but not sufficient condition to achieve that $r_{-1}^{MP} < r_0^{MP}$.

tations, it implies that $\pi_2^e = \pi_1 = \pi^T$ again. Having achieved this outcome, then, in $t = 1$ the CB will reduce the real interest rate, i.e., set $r_1^{MP} < r_0^{MP}$. But, from Equation (62) it follows the CB sets the nominal interest-rate i_1^{MP} such that the following holds:

$$\frac{1 + i_1^{MP}}{1 + \pi_2^e} - 1 < \frac{1 + i_0^{MP}}{1 + \pi_1^e} - 1$$
$$\frac{1 + i_1^{MP}}{1 + \pi^T} < \frac{1 + i_0^{MP}}{1 + \pi^T + \alpha A}$$

which, after some algebra, boils down to:

$$i_1^{MP} < \frac{i_0^{MP}(1 + \pi^T) - \alpha A}{1 + \pi^T + \alpha A}$$

But note that:

$$\frac{i_0^{MP}(1 + \pi^T) - \alpha A}{1 + \pi^T + \alpha A} < i_0^{MP}$$

Therefore, the qualitative result obtained before, namely, $i_1^{MP} < i_0^{MP}$, is not altered.[24]

Given its forward-looking and highly competent, the CB will reduce r_1 rightly forecasting (since $t = 1$) (a) that next period's PC will be given that $\pi_2 = \pi^T + \alpha(Y_2 - Y_{e,2})$, and (b) the exact amount in which r_1^{MP} has to be below r_0^{MP} to achieve that $Y_2 - Y_{e,2} = 0$.

Summarizing, the economy ends up $t = 1$ with $Y_1 < Y_0$ and $Y_{e,0} < Y_{e,1}$ such

[24] Actually, it verifies that $i_1^{MP} < i_0^{MP}$ is necessary but not sufficient condition to achieve that $r_1^{MP} < r_0^{MP}$.

that (a) $Y_1 - Y_{e,1} = -A < 0$, (b) $\pi_1 = \pi^T$, and (c) $r_1^{MP} < r_0^{MP}$; the latter in turn implies that $i_1^{MP} < i_0^{MP}$, as verified.

- The new MRE ($t = 2$ and thereafter)

As consequence of last-period's interest-rate reduction, in $t = 2$ aggregate demand is boosted, i.e., $Y_1 < Y_2$, and equilibrium output reduced, i.e., $Y_{e,2} < Y_{e,1}$. Furthermore (because of the high rationality and capabilities of the CB) these effects are such that they exactly close the output gap, i.e., $Y_2 - Y_{e,2} = 0$. Consequently, the inflation rate will be:

$$\begin{aligned} \pi_2 &= \pi^T \alpha(Y_2 - Y_{e,2}) \\ &= \pi^T + \alpha(0) \\ &= \pi^T, \end{aligned}$$

that is, it'll remain at the CB's target; furthermore, now it's constant as well: $\pi^T = \pi_2 = \pi_0$. Given the adaptive inflation expectations, it implies that $\pi_3^e = \pi_2 = \pi^T$ again. Having achieved this outcome, the CB will no longer alter the real interest rate, i.e., it'll set $r_2^{MP} = r_1^{MP}$. But, from Equation (62) it follows the CB sets the nominal interest-rate i_2^{MP} such that the following holds:

$$\frac{1 + i_2^{MP}}{1 + \pi_3^e} - 1 = \frac{1 + i_1^{MP}}{1 + \pi_2^e} - 1$$
$$\frac{1 + i_2^{MP}}{1 + \pi^T} = \frac{1 + i_1^{MP}}{1 + \pi^T}$$

which, of course, implies that:

$$i_2^{MP} = i_1^{MP};$$

which is exactly the same result obtained before. Summarizing, the economy ends up $t = 2$ with $Y_1 < Y_2$ and $Y_{e,2} < Y_{e,1}$ such that (a) $Y_2 - Y_{e,2} = 0$, (b) $\pi_2 = \pi^T$, and (c) $r_2^{MP} = r_1^{MP}$; the latter now implies that $i_2^{MP} = i_1^{MP}$. And this is the new MRE configuration.

ii. ◦ The shock's period ($t = 0$)

In this case, the reduction in p_o for sure creates a negative output gap in the shock's period; say it's of size $-A < 0$ (that is, $Y_0 - Y_{e,o} = -A < 0$). This negative output gap will puts the inflation rate below the CB's target inflation rate during the shock's period. Specifically, from Equation (60) it follows that:

$$\begin{aligned}\pi_0 &= \pi^T + \alpha(Y_0 - Y_{e,0}) \\ &= \pi^T - \alpha A\end{aligned}$$

Given the adaptive inflation expectations, it implies that $\pi_1^e = \pi_0 = \pi^T - \alpha A$. Then, as response, the CB will carry out an expansionary monetary policy, which in this period means it'll reduce the real interest rate. i.e., set $r_0^{MP} < r_{-1}^{MP}$. But, from Equation (62) it follows the CB sets the nominal interest-rate

i_0^{MP} such that the following holds:

$$\frac{1+i_0^{MP}}{1+\pi_1^e} - 1 < \frac{1+i_{-1}^{MP}}{1+\pi_0^e} - 1$$
$$\frac{1+i_0^{MP}}{1+\pi^T-\alpha A} < \frac{1+i_{-1}^{MP}}{1+\pi^T}$$

which, after some algebra, boils down to:

$$i_0^{MP} < i_{-1}^{MP} - \frac{(1+i_{-1}^{MP})\alpha A}{1+\pi^T}$$

But note that:

$$i_{-1}^{MP} - \frac{(1+i_{-1}^{MP})\alpha A}{1+\pi^T} < i_{-1}^{MP}$$

Therefore, the qualitative result obtained before, namely, $i_0^{MP} < i_{-1}^{MP}$, is not altered.[25]

Given its forward-looking and highly competent, the CB will reduce r_0 rightly forecasting, since this $t = 0$, (a) that next period's PC will be given that $\pi_1 = \pi^T + \alpha A + \alpha(Y_1 - Y_{e,1})$, and (b) the exact amount in which r_0^{MP} has to be below r_{-1}^{MP} to achieve that $0 < A = Y_1 - Y_{e,1}$.

Summarizing, the economy ends up $t = 0$ with $Y_{e,0} < Y_{e,-1}$ and $Y_0 < Y_{-1}$ so that (a) $Y_0 - Y_{e,0} = -A < 0$, (b) $\pi_0 < \pi^T$, and (c) $r_{-1}^{MP} < r_0^{MP}$; the latter in turn implies that $i_0^{MP} < i_{-1}^{MP}$, as verified.

- Following to the shock's period ($t = 1$)

[25] Actually, it verifies that $i_0^{MP} < i_{-1}^{MP}$ is necessary but not sufficient condition to achieve that $r_0^{MP} < r_{-1}^{MP}$.

As consequence of the last-period's interest-rate reduction, in $t = 1$ aggregate demand increases, i.e., $Y_0 < Y_1$, and equilibrium output decreases, i.e., $Y_{e,1} < Y_{e,0}$. Furthermore (because of the high rationality and capabilities of the CB) these effects are such that they create a positive output gap which is equal, in absolute value, to the last-period's negative gap, i.e., $0 < A = Y_1 - Y_{e,1}$. Consequently, the inflation rate will be:

$$\begin{aligned}\pi_1 &= \pi^T - \alpha A + \alpha(Y_1 - Y_{e,1}) \\ &= \pi^T - \alpha A + \alpha A \\ &= \pi^T,\end{aligned}$$

that is, it aligns with the the CB's target. Given the adaptive inflation expectations, it implies that $\pi_2^e = \pi_1 = \pi^T$ again. Having achieved this outcome, then, in $t = 1$ the CB will increase the real interest rate, i.e., set $r_0^{MP} < r_1^{MP}$. But, from Equation (62) it follows the CB sets the nominal interest-rate i_1^{MP} such that:

$$\frac{1 + i_0^{MP}}{1 + \pi_1^e} - 1 < \frac{1 + i_1^{MP}}{1 + \pi_2^e} - 1$$
$$\frac{1 + i_0^{MP}}{1 + \pi^T - \alpha A} < \frac{1 + i_1^{MP}}{1 + \pi^T}$$

which, after some algebra, boils down to:

$$\frac{i_0^{MP}(1+\pi^T)+\alpha A}{1+\pi^T-\alpha A} < i_1^{MP}$$

But note that:

$$i_0^{MP} < \frac{i_0^{MP}(1+\pi^T)+\alpha A}{1+\pi^T-\alpha A}$$

Therefore, the qualitative result obtained before, namely, $i_0^{MP} < i_1^{MP}$, is not altered.[26]

Given its forward-looking and highly competent, the CB will increase r_1 rightly forecasting (since $t = 1$) (a) that next period's PC will be given that $\pi_2 = \pi^T + \alpha(Y_2 - Y_{e,2})$, and (b) the exact amount in which r_1^{MP} has to be above r_0^{MP} to achieve that $Y_2 - Y_{e,2} = 0$.

Summarizing, the economy ends up $t = 1$ with $Y_0 < Y_1$ and $Y_{e,1} < Y_{e,0}$ such that (a) $0 < A = Y_1 - Y_{e,1}$, (b) $\pi_1 = \pi^T$, and (c) $r_0^{MP} < r_1^{MP}$; the latter in turn implies that $i_0^{MP} < i_1^{MP}$, as verified.

- The new MRE ($t = 2$ and thereafter)

  As consequence of last-period's interest-rate increase, in $t = 2$ aggregate demand is reduced, i.e., $Y_2 < Y_1$, and equilibrium output increases, i.e., $Y_{e,1} < Y_{e,2}$. Furthermore (because of the high rationality and capabilities of the CB) these effects are such that they exactly close the output gap, i.e., $Y_2 - Y_{e,2} = 0$.

[26]Actually, it verifies that $i_0^{MP} < i_1^{MP}$ is necessary but not sufficient condition to achieve that $r_0^{MP} < r_1^{MP}$.

Consequently, the inflation rate will be:

$$\pi_2 = \pi^T \alpha(Y_2 - Y_{e,2})$$
$$= \pi^T + \alpha(0)$$
$$= \pi^T,$$

that is, it'll remain at the CB's target; furthermore, now it's constant as well: $\pi^T = \pi_2 = \pi_0$. Given the adaptive inflation expectations, it implies that $\pi_3^e = \pi_2 = \pi^T$ again. Having achieved this outcome, the CB will no longer alter the real interest rate, i.e., it'll set $r_2^{MP} = r_1^{MP}$. But, from Equation (62) it follows the CB sets the nominal interest-rate i_2^{MP} such that the following holds:

$$\frac{1 + i_2^{MP}}{1 + \pi_3^e} - 1 = \frac{1 + i_1^{MP}}{1 + \pi_2^e} - 1$$
$$\frac{1 + i_2^{MP}}{1 + \pi^T} = \frac{1 + i_1^{MP}}{1 + \pi^T}$$

which, of course, implies that:

$$i_2^{MP} = i_1^{MP}$$

which is exactly the same result obtained before.

Summarizing, the economy ends up $t = 2$ with $Y_2 < Y_1$ and $Y_{e,1} < Y_{e,2}$ such that (a) $Y_2 - Y_{e,2} = 0$, (b) $\pi_2 = \pi^T$, and (c) $r_2^{MP} = r_1^{MP}$; the latter now implies that $i_2^{MP} = i_1^{MP}$. And this is the new MRE configuration.

21. To keep the analysis as simple as possible, in Gómez-Ramírez and Quintero Otero (2026) analysis of a reduction in p_o it is assumed that, after a shock happening in some $t = 0$, the CB is capable to (a) bring back the inflation rate to its target inflation rate in the following to the shock period, that is, in $t = 1$ (i.e., that $\pi_1 = \pi^T$ again), which is the fastest possible and simplest our model allows;and (b) close the output gap in the following to it period, that is, in $t = 2$ (i.e., $Y_2 - Y_{e,2} = 0$, which is, again, the fastest and simplest the model allows). In reality, however, as you may have listened to, stabilizing Central Bank's interventions may take (many) more periods and complications. In this exercise you will dig deeper into this certainly challenging issue. However, as I hope you will realize, the exercise is ordered in a way that already guides you about one way in which you could address this thorny issues.

    Therefore, assume that, initially ($t = -1$), the economy is at some MRE, in which (a) $i_{-1} = i_{-1}^{MP}$, (b) $Y_{-1} = Y_{e,-1}$, and (c) $\pi_{-1} = \pi^T$. But, then, in $t = 0$, p_o experiences an exogenous one-time and permanent reduction: $p_{o,k} < p_{o,-1}$ for $0 \leq k$. Assume this reduction does not reduce equilibrium aggregate demand or it reduces it but not as much as it reduces equilibrium output, so that (as at this stage of this exercises list you may have already heard a couple of times!) a positive gap is created at $t = 0$; specifically, let it be of size $0 < A$, i.e., $0 < A = \alpha(Y_0 - Y_{e,0})$. Then, using Gómez-Ramírez and Quintero Otero (2026)'s model, do/answer all the following:

i. Calculate the negative output gap that the CB ultimately needs to achieve in order to, at some undetermined period j, align the inflation rate with $\pi_{-1} = \pi^T$, i.e., to achieve that $\pi_j = \pi^T$ again.

ii. If it's the case that, by setting i^{MP}_{k-1} at any period $t = k - 1$, then every following to it period, $t = k$, the monetary authority is able to obtain only an amount $0 < \gamma_k < 1$ of the negative gap you found in item (i), what would be the inflation rate at $t = 1$? At $t = 2$? At $t = 3$? Therefore, what has to be case to achieve that, at the undetermined period $t = j$, $\pi_j = \pi^T$ finally occurs? For the sake of the analysis, assume that $5 < j$ although, of course, it does not necessarily have to be the case.[27]

iii. (This item is somewhat a digression but it highlights an interesting and likely important issue) In item (ii) I told you to assume that $0 < \gamma_k < 1$ every period $t = k$. Differently, what would it mean to assume that, for some reason (say CB's decision-making is not perfect), at one (or more than one) period $t = k$ it happens that $1 < \gamma_k$? And, if the latter happened, how would it have to be γ at another $t \neq k$ period (or more periods) to achieve that π^T? Explain what it means.

iv. Now consider the situation in which the monetary policymaker has already been able to align $\pi_j = \pi^T$. Then, what would it have to achieve in $t = j+1$ (by setting i^{MP}_j in $t = j$, as you know) in order to achieve that $\pi_{j+1} = \pi^T$ remains the case?

Note: If in item (v) it helps you to set $j = 1$ (as if, once $\pi = \pi^T$ is the case again, time

[27] As you'll realize when solving the exercise, this assumption means that $\gamma_1 \neq 1$, $\gamma_1 + \gamma_1 \neq 1$, $\gamma_1 + \gamma_2 + \gamma_3 \neq 1$, $\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 \neq 1$, and $\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 \neq 1$.

started again) do it. However, don't overlook that the complete stabilization takes the j periods you dealt with in item (ii) and the h periods you will deal with in item (v).

v. Consider the case in which, although it wishes to, the Central Bank cannot close the output gap in the $t = j + 1$ period and, instead, each period $t = j + k$ (by setting the interest rate at $t = j + k - 1$, of course) it can only obtain that $Y_{j+k} = \delta_{j+k} Y_{e,j+k}$ with $\delta_{j+1} \neq 1$. Then, what would be the inflation rate at $t = j + 1$? At $t = j + 2$? At $t = j + 3$? Therefore, what has to be the case to achieve that, at the indeterminate period $t = j + h$, $\pi_{j+h} = \pi^T$ finally occurs? For the sake of the analysis, assume that $5 < h$ although, of course, it does not have to be the case.[28]

vi. This item is about an interesting characteristic of the stabilization process that, because inflation expectations are adaptive and the monetary authority is unable to achieve that $Y_{j+1} - Y_{e,j+1} = 0$ in the very $j+1$ period, must ensue after $\pi_j = \pi^T$ is achieved.[29] Thus, you may have noted that, in item (v), the only restriction we

[28] As you'll realize when solving the exercise, this assumption means that $\delta_{j+1} + \delta_{j+2} \neq 2$, $\delta_{j+1} + \delta_{j+2} + \delta_{j+3} \neq 3$, $\delta_{j+1} + \delta_{j+2} + \delta_{j+3} + \delta_{j+4} \neq 4$, and $\delta_{j+1} + \delta_{j+2} + \delta_{j+3} + \delta_{j+4} + \delta_{j+5} \neq 5$.

[29] When solving this exercise, you will realize that the inflation inertia embedded in model –coming from the adaptive inflation expectations' pheature– together with the CB's inability to achieve that $Y_{j+1} = Y_{e,j+1}$ (in the very following period, $t = j + 1$) imply that, once $\pi_j = \pi^T$ is achieved, the stabilization process will not be smooth but it will imply at least one more boom-and-bust or bust-and-boom cycle around the new equilibrium output.

It's worth mentioning that this issue does not arise in Carlin and Soskice (2006, 2015)'s 3-equation model, because it implies the following interest-rate rule (Carlin and Soskice, 2015, pp. 477-478):

$$r_t - r_s = \frac{\alpha\beta}{a(1 + \alpha^2\beta)}(\pi_t - \pi^T) \tag{63}$$

In Equation 63 r_s denotes the stabilizing interest-rate (which closes the output gap), $0 < a$ a parameter that measures how much $Y - Y_e$ (inversely) reacts to $r - r_s$ differences, $0 < \alpha$ the slope of the Phillips curve, $0 < \beta$ a parameter that measures the monetary authority aversion to inflation, and $0 < \pi^T$ the monetary authority's inflation target rate. As you could see, Equation (63) implies that, if $\pi_j = \pi^T$ is achieved, then, without further delay, the monetary authority does have the ability to achieve that $r_j = r_s$, i.e., does have imposed on the values of δ_{j+k} is that the first one ($k = 1$) is not equal to 1 (that is, $\delta_{j+1} \neq 1$) and, furthermore, we did not impose that $\delta_{j+k} < 1$ or that $1 < \delta_{j+k}$ for any $1 \leq k$ (including $k = 1$). What is the rationale of this general absence of restrictions? You can answer this question by, first, finding what would happen to the inflation rate if:

a. $\delta_{j+k} < 1$ for every $1 \leq k$ were the case;

b. $1 < \delta_{j+k}$ for every $1 \leq k$ were the case;

and, second, explain more in detail the scenarios which would follow if we assumed that (and, of course, also assuming that the monetary policymaker wants to finally close the output gap and align the inflation rate with its target):

a. $\delta_{j+1} < 1$;

b. $1 < \delta_{j+1}$.

Answers:

i. Note, first, that, from the fact that the positive output gap at $t = 0$ is $0 < A = Y_0 - Y_{e,0}$, it follows that $\pi_0 = \pi^T + \alpha(Y_1 - Y_{e,0}) = \pi^T + \alpha A$. Therefore, to eventually align the inflation rate to π^T at the undetermined period $t = j$, the monetary authority ultimately needs to achieve that:

$$\alpha(Y_0 - Y_{e,0}) + \alpha(Y_j - Y_{e,j}) = 0$$

the ability to close the output gap without further delay. But this is precisely the assumption that item (vi) of this exercise (21) drops.

(equivalently, $\alpha(Y_0 - Y_{e,0})\sigma_{N,0} - \alpha(Y_{e,j} - Y_j) = 0$). Solving the latter for $Y_j - Y_{e,j}$, it follows that the negative output gap the monetary authority ultimately needs to achieve is:

$$\begin{aligned} Y_j - Y_{e,j} &= -(Y_0 - Y_{e,0}) \\ &= -A \end{aligned}$$

(equivalently, $Y_{e,j} - Y_j = Y_0 - Y_{e,0} = A$).

ii. By setting i^{MP}_{k-1} at $t = k - 1$, each period $t = k$ the CB could achieve an amount $0 < \gamma_k < 1$ of the desired negative gap obtained in item (i). Therefore, by setting i^{MP}_0 in $t = 0$, by $t = 1$ it will achieve that:

$$\begin{aligned} Y_1 - Y_{e,1} &= -\gamma_1(Y_0 - Y_{e,0}) \\ &= -\gamma_1 A \end{aligned}$$

It follows that:

$$\begin{aligned} \pi_1 &= \pi_0 + \alpha(Y_1 - Y_{e,1}) \\ &= \pi^T + \alpha A + \alpha(-\gamma_1 A) \\ &= \pi^T + \alpha(1 - \gamma_1)A \end{aligned}$$

(Note, because it will help for item iii's answer, that $\pi_1 \lesseqgtr \pi^T \iff 1 \lesseqgtr \gamma_1$).

Next, by setting i_1^{MP} in $t = 1$, by $t = 2$ the CB will achieve that:

$$
\begin{aligned}
Y_2 - Y_{e,2} = & -\gamma_2(Y_0 - Y_{e,0}) \\
= & -\gamma_2 A
\end{aligned}
$$

It follows that:

$$
\begin{aligned}
\pi_2 &= \pi_1 + \alpha(Y_2 - Y_{e,2}) \\
&= \pi^T + \alpha(1-\gamma_1)A + \alpha(-\gamma_2 A) \\
&= \pi^T + \alpha(1-\gamma_1-\gamma_2)A
\end{aligned}
$$

(Note, because it will help for item iii's answer, that $\pi_2 \lesseqgtr \pi^T \iff 1 \lesseqgtr \gamma_1 + \gamma_2$).

Next, by setting i_2^{MP} in $t = 2$, by $t = 3$ the CB will achieve that:

$$
\begin{aligned}
Y_3 - Y_{e,3} = & -\gamma_3(Y_0 - Y_{e,0}) \\
= & -\gamma_3 A
\end{aligned}
$$

It follows that:

$$
\begin{aligned}
\pi_3 &= \pi_2 + \alpha(Y_3 - Y_{e,3}) \\
&= \pi^T + \alpha(1-\gamma_1-\gamma_2)A + \alpha(-\gamma_3 A) \\
&= \pi^T + \alpha(1-\gamma_1-\gamma_2-\gamma_3)A
\end{aligned}
$$

(Note, because it will help for item iii's answer, that $\pi_3 \lesseqgtr \pi^T \iff 1 \lesseqgtr \gamma_1 + \gamma_2 + \gamma_3$). Therefore, you can notice that (under the assumption that $0 < \gamma_k < 1$) each period the inflation rate is closer to π^T. This process will keep going until, by setting i_{j-1}^{MP} at some undetermined period $t = j-1$, then by the also undetermined period $t = j$ the CB achieves the negative output gap:

$$
\begin{aligned}
Y_j - Y_{e,j} &= -\gamma_j(Y_0 - Y_{e,0}) \\
&= -\gamma_j A
\end{aligned}
$$

It, in turn, achieves that:

$$
\begin{aligned}
\pi_j &= \pi_{j-1} + \alpha(Y_j - Y_{e,j}) \\
&= \pi^T + \alpha(1 - \gamma_1 - \gamma_2 - \gamma_3 - \gamma_4 - ... - \gamma_{j-1})A + \alpha(-\gamma_j A) \\
&= \pi^T + \alpha(1 - \gamma_1 - \gamma_2 - \gamma_3 - \gamma_4 - ... - \gamma_{j-1} - \gamma_j)A \\
&= \pi^T + \alpha\left(1 - \sum_{k=1}^{j} \gamma_k\right) A \\
&= \pi^T
\end{aligned}
$$

Therefore, you can see that, for $\pi_j = \pi^T$ to finally be the case at the undetermined period $t = j$, that

$$
1 = \sum_{k=1}^{j} \gamma_k
$$

has to be the case.

iii. Note that:

$$\pi_j \lesseqqgtr \pi^T \iff 1 \lesseqqgtr \sum_{k=1}^{j} \gamma_k.$$

Therefore, if for some reason at one (or more than one) period $t = k$ it occurred that $1 < \gamma_k$, then $\pi_j < \pi^T$, that is, the CB would be depressing the economy more than it needs to lead it back to π^T (and, thus, would be yielding an smaller π_j rate). To undo this, at another period $t \neq k$ (or at more than one), the CB will have to achieve that $\delta_{\neq k} < 0$; because only then $\sum_{k=1}^{j} \gamma_k = 1$ will nevertheless hold. It means that, to undo the over-depression it created at period $t = k$ (or more periods), the CB would have to create a positive output gap at another period $t \neq k$ (or more periods).

iv. If $\pi_j = \pi^T$ has been achieved, then

$$\begin{aligned}\pi_{j+1} &= \pi_j + \alpha(Y_{j+1} - Y_{e,j+1}) \\ &= \pi^T + \alpha(Y_{j+1} - Y_{e,j+1})\end{aligned}$$

Therefore, to achieve that $\pi_{j+1} = \pi^T$ remains, the CB has to achieve that $Y_{j+1} - Y_{e,j+1}$, that is, it has to close the output gap at period $t = j + 1$.

v. By setting, at period $t = j + (k - 1)$, the interest rate $i^{MP}_{j+(k-1)}$ (for $0 \leq k$, of course), then each period $t = j + k$ the monetary policymaker can only achieve that $Y_{j+k} = \delta_{j+k} Y_{e,j+k}$ with $\delta_{j+1} \neq 1$ (that is, the CB cannot close the output gap in period $t = j + 1$). Therefore, by setting i^{MP}_j in $t = j$, then by $t = j + 1$ the CB will achieve that:

$$Y_{j+1} - Y_{e,j+1} = \delta_{j+1} Y_{e,j+1} - Y_{e,j+1}$$
$$= (\delta_{j+1} - 1) Y_{e,j+1}$$

(note that $Y_{j+1} - Y_{e,j+1} \lesseqgtr 0 \iff \delta_{j+1} \lesseqgtr 1$). It follows that:

$$\pi_{j+1} = \pi_j + \alpha(Y_{j+1} - Y_{e,j+1})$$
$$= \pi^T + \alpha(\delta_{j+1} - 1) Y_{e,j+1}$$

(Note that $\pi_{j+1} \lesseqgtr \pi^T \iff \delta_{j+1} \lesseqgtr 1$). Next, by setting i_{j+1}^{MP} in $t = j+1$, then by $t = j+2$ the CB will achieve that:

$$Y_{j+2} - Y_{e,j+2} = \delta_{j+2} Y_{e,j+2} - Y_e^{new}$$
$$= (\delta_{j+2} - 1) Y_{e,j+2}$$

(note that $Y_{j+2} - Y_{e,j+2} \lesseqgtr 0 \iff \delta_{j+2} \lesseqgtr 1$). It follows that:

$$\pi_{j+2} = \pi_{j+1} + \alpha(Y_{j+2} - Y_{e,j+2})$$
$$= \pi^T + \alpha(\delta_{j+1} - 1) Y_{e,j+1} + \alpha(\delta_{j+2} - 1) Y_{e,j+2}$$
$$= \pi^T + \alpha \left((\delta_{j+1} - 1) Y_{e,j+1} + (\delta_{j+2} - 1) Y_{e,j+2} \right)$$

(Note that $\pi_{j+2} \lesseqgtr \pi^T \iff \delta_{j+1} \lesseqgtr 1$ & $\delta_{j+2} \lesseqgtr 1$).

Next, by setting i_{j+2}^{MP} in $t = j + 2$, then by $t = j + 3$ the CB will achieve that:

$$
\begin{aligned}
Y_{j+3} - Y_{e,j+3} &= \delta_{j+3} Y_{e,j+3} - Y_{e,j+3} \\
&= (\delta_{j+3} - 1) Y_{e,j+3}
\end{aligned}
$$

(note that $Y_{j+3} - Y_{e,j+3} \lesseqgtr 0 \iff \delta_{j+3} \lesseqgtr 1$). It follows that:

$$
\begin{aligned}
\pi_{j+3} &= \pi_{j+2} + \alpha(Y_{j+3} - Y_{e,j+3}) \\
&= \pi^T + \alpha(\delta_{j+1} - 1)Y_{e,j+1} + \alpha(\delta_{j+2} - 1)Y_{e,j+2} + \alpha(\delta_{j+3} - 1)Y_{e,j+3} \\
&= \pi^T + \alpha\left((\delta_{j+1} - 1)Y_{e,j+1} + (\delta_{j+2} - 1)Y_{e,j+2} + (\delta_{j+3} - 1)Y_{e,j+3}\right)
\end{aligned}
$$

(Note that $\pi_{j+3} \lesseqgtr \pi^T \iff \delta_{j+1} \lesseqgtr 1$ & $\delta_{j+2} \lesseqgtr 1$ & $\delta_{j+3} \lesseqgtr 1$).

Next, by setting i_{j+3}^{MP} in $t = j + 3$, then by $t = j + 4$ the CB will achieve that:

$$
\begin{aligned}
Y_{j+4} - Y_{e,j+4} &= \delta_{j+4} Y_{e,j+4} - Y_{e,j+4} \\
&= (\delta_{j+4} - 1) Y_{e,j+4}
\end{aligned}
$$

(note that $Y_{j+4} - Y_{e,j+4} \lesseqgtr 0 \iff \delta_{j+4} \lesseqgtr 1$). It follows that:

$$\begin{aligned}
\pi_{j+4} &= \pi_{j+3} + \alpha(Y_{j+4} - Y_{e,j+4}) \\
&= \pi^T + \alpha(\delta_{j+1} - 1)Y_{e,j+1} + \alpha(\delta_{j+2} - 1)Y_{e,j+2} + \alpha(\delta_{j+3} - 1)Y_{e,j+3} \\
&\quad +(\delta_{j+4} - 1)Y_{e,j+4} \\
&= \pi^T + \alpha((\delta_{j+1} - 1)Y_{e,j+1} + (\delta_{j+2} - 1)Y_{e,j+2} + (\delta_{j+3} - 1)Y_{e,j+3} \\
&\quad +(\delta_{j+4} - 1)Y_{e,j+4})
\end{aligned}$$

(Note that $\pi_{j+4} \lesseqgtr \pi^T \iff \delta_{j+1} \lesseqgtr 1 \;\&\; \delta_{j+2} \lesseqgtr 1 \;\&\; \delta_{j+3} \lesseqgtr 1 \;\&\; \delta_{j+4} \lesseqgtr 1$). This process will keep going until, by setting i^{MP}_{j+h-1} at some undetermined period $t = j + h - 1$, then by the also undetermined period $t = j + h$ the CB achieves the output gap:

$$\begin{aligned}
Y_{j+h} - Y_{e,j+h} &= \delta_{j+h}Y_{e,j+h} - Y_{e,j+h} \\
&= (\delta_{j+h} - 1)Y_{e,j+h}
\end{aligned}$$

(note that $Y_{j+h} - Y_{e,j+h} \lesseqgtr 0 \iff \delta_{j+h} \lesseqgtr 1$). It, in turn, achieves that:

$$\begin{aligned}
\pi_{j+h} &= \pi_{j+h-1} + \alpha(Y_{j+h} - Y_{e,j+h}) \\
&= \pi^T + \alpha(\delta_{j+1} - 1)Y_{e,j+1} + \alpha(\delta_{j+2} - 1)Y_{e,j+2} + \alpha(\delta_{j+3} - 1)Y_{e,j+3} \\
&\quad + (\delta_{j+4} - 1)Y_{e,j+4} + ... + \alpha(\delta_{j+h-1} - 1)Y_{e,j+h-1} + \alpha(\delta_{j+h} - 1)Y_{e,j+(h)} \\
&= \pi^T + \alpha((\delta_{j+1} - 1)Y_{e,j+1} + (\delta_{j+2} - 1)Y_{e,j+2} + (\delta_{j+3} - 1)Y_{e,j+3} \\
&\quad + (\delta_{j+4} - 1)Y_{e,j+4} + (\delta_{j+h-1} - 1)Y_{e,j+h-1} + (\delta_{j+h} - 1)Y_{e,j+(h)}) \\
&= \pi^T + \alpha\left(\sum_{k=1}^{h} (\delta_{j+k} - 1)\, Y_{e,j+k}\right) \\
&= \pi^T
\end{aligned}$$

Therefore, you can seen that, for $\pi_{j+h} = \pi^T$ to finally be the case at the undetermined period $t = j + h$, that

$$\sum_{k=1}^{h} (\delta_{j+k} - 1)\, Y_{e,j+k} = 0$$

has to be the case.

vi. a. Consider the case in which we had assumed that $\delta_{j+k} < 1$ for every $t = j + k$ period. In this scenario, after h periods, $\sum_{k=1}^{h} (\delta_{j+k} - 1)\, Y_{e,j+k} < 0$. But, then, it will be the case that $\pi_{j+h} < \pi^T$; specifically:

$$\pi_{j+h} = \pi^T - \alpha\left|\sum_{k=1}^{h} (\delta_{j+k} - 1)\, Y_{e,j+k}\right|$$

In other words, in this scenario, each period the monetary authority would be creating decreasing inflation (recall that $\delta_{j+k} < 1$ implies a negative output gap at such $t = j + k$ period: $Y_{j+k} - Y_{j,k} < 0$).

b. Now consider the scenario in which we had assumed that $1 < \delta_{j+k}$ for every $t = j + k$ period. In this case, after h periods, $0 < \sum_{k=1}^{h} (\delta_{j+k} - 1) Y_{e,j+k}$. But, then, it will be the case that $\pi^T < \pi_{j+h}$; specifically:

$$\pi_{j+h} = \pi^T + \alpha \left(\sum_{k=1}^{h} (\delta_{j+k} - 1) Y_{e,j+k} \right)$$

In other words, in this scenario, each period the monetary authority would be creating increasing inflation each period (recall that $1 < \delta_{j+k}$ implies a positive output gap at such $t = j + k$ period: $0 < Y_{j+k} - Y_{e,j+k}$).

Therefore, you can now see that, from the fact that $\delta_{j+1} \neq 1$, it follows that the only way in which $\sum_{k=1}^{h} (\delta_{j+k} - 1) Y_{e,j+k} = 0$ is the case (so that actually $\pi_{j+h} = \pi^T$ is the case) is that at least one subsequent δ_{j+k} ($k \neq 1$) is greater or smaller than 1, depending on each of the following two scenarios:

a. If $\delta_{j+1} < 1$ (i.e, the monetary authority produced a negative output gap in $t = j + 1$) at least one another future $1 < \delta_{j+k}$ (i.e, the monetary authority will have to produce a positive output gap in at least one subsequent period). Therefore, in this scenario the economy experiences at least another bust-and-boom cycle before finally closing the output gap and achieving that the inflation rate is π^T.

b. If $1 < \delta_{j+1}$ (i.e., the monetary authority produced a positive output gap in $t = j + 1$) at least one another future $1 < \delta_{j+k}$ (i.e., the monetary authority will have to produce a negative output gap in at least one subsequent period). Therefore, in this scenario, the economy experiences at least another boom-and-bust cycle before finally closing the output gap and achieving that the inflation rate is π^T.

22. In Gómez-Ramírez and Quintero Otero (2026) a permanent reduction in p_o is examined. However, examining a temporary reduction is very important in its own. Therefore, this exercise will make you go over this topic. Thus, assume that, initially ($t = -1$), the economy is at some MRE, in which (a) $i_{-1} = i^{MP}_{-1}$, (b) $Y_{-1} = Y_{e,-1}$, and (c) $\pi_{-1} = \pi^T$. But, then, in $t = 0$, p_o experiences an exogenous one-time and one-period only reduction, that is:

  - in $t = 0$ it occurs that the oil price decreases: $p_{o,0} < p_{o,-1}$;
  - but, then, in $t = 1$ the oil prices increases back to its original value and remains that in every subsequent period: $p_{o,-1} = p_{o,1} = p_{o,2} = p_{o,3} = \ldots$ (more succintly: $p_{o,-1} = p_{o,k}$ for $1 \leq k$).

To simplify it and make it easier to compare with Gómez-Ramírez and Quintero Otero (2026)'s subsections 4.1–4.5 analysis, assume as well that (a) the reduction of p_0 in $t = 0$ leaves equilibrium aggregate demand overall unaltered (that is, $Y_{-1} = Y_0$), so that a positive gap is created in $t = 0$; specifically, let it be of size $0 < A$, that is, $0 < A = \alpha(Y_0 - Y_{e,0})$; and (b) if not accompanied by CB's intervention, the return of p_0 to its original value that occurs in $t = 1$ would, again, leave aggregate demand overall unchanged (that is, $Y_0 = Y_1$ would be the case), and increase equilibrium output back to its original MRE value as well (that is, $Y_{e,1} = Y_{e,0}$ would be the case), which also implies that the output gap would be closed, that is, $\alpha(Y_1 - Y_{e,1}) = \alpha(Y_{-1} - Y_{e,-1}) = 0$ would be the case. Then, do/answer all the following (of course, using Gómez-Ramírez and Quintero Otero, 2026, model):

i. Assuming the CB does not intervene (that is, $i_{-1}^{MP} = i_k^{MP}$ remains the case for any $0 \leq k$), examine what would happen, period by period, subsequently.

ii. Comparing it with the scenario of a permanent reduction in p_0 that creates a positive output gap but is not accompanied by CB intervention (exercise 17 above), what is, then, the main takeaways of the analysis you carried out in item (i)?

iii. Assuming the CB does intervene, examine what would happen, period by period, subsequently. Assume that the CB is forward-looking and highly competent so that it's rightly understands the shock is temporary, not permanent, and is capable of fully bringing back the economy toward an MRE configuration in $t = 2$.

iv. Comparing it with the scenario of a permanent reduction in p_0 that it is accompanied by the intervention of a forward-looking and highly capable CB (which is the case in Gómez-Ramírez and Quintero Otero, 2026, subsections

4.1–4.5), what is, then, the main takeaways of the analysis you carried out in item (iii)?

Answers:

i. ◦ The shock's period ($t = 0$)

To begin with, the reduction in p_o reduces Y_e, that is, $Y_{e,0} < Y_{e,-1}$. And, given we assumed it leaves aggregate demand overall unchanged, that is, $Y_0 = Y_{-1}$, then for sure a positive output gap arises in the shock's period; it's of size $0 < A$, that is, $0 < A = \alpha(Y_0 - Y_{e,0})$. This positive output gap will increase the inflation rate above the CB's target during the shock's period. Specifically, from Equation (60) it follows that:

$$\begin{aligned}\pi_0 &= \pi^T + \alpha(Y_0 - Y_{e,0}) \\ &= \pi^T + A \\ &> \pi^T\end{aligned}$$

In turn, given the CB does not intervene, then $i_{-1}^{MP} = i_0^{MP}$ will be the case.

Summarizing, the economy ends up $t = 0$ with $Y_{e,0} < Y_{e,-1}$ and $Y_{-1} = Y_0$ such that (a) $0 < A = Y_0 - Y_{e,0}$ and (b) $\pi^T < \pi_0$; furthermore, (c) $i_{-1}^{MP} = i_0^{MP}$.

◦ Period in which the temporary shock is undone ($t = 1$)

We assumed that, in the absence of CB intervention, the return of the oil price to its original value ($p_1 = p_{-1}$ again), both increases Y_e ot its original MRE level, that is, $Y_{e,0} < Y_{e,1} = Y_{e,-1}$ and also leaves aggregate demand overall the same, that is, $Y_1 = Y_0$. Therefore, the output gap will be closed, that is, $Y_1 - Y_{e,1} = 0$. Consequently, the inflation rate will be:

$$\begin{aligned}
\pi_1 &= \pi^T + A + \alpha(Y_1 - Y_{e,1}) \\
&= \pi^T + A + \alpha(0) \\
&= \pi^T + A \\
&= \pi_0
\end{aligned}$$

that is, the inflation rate will not further increase. In turn, given the CB does not intervene, then $i_0^{MP} = i_1^{MP}$ will be the case.

Summarizing, the economy ends up $t = 1$ with $Y_1 = Y_0$ and $Y_0 < Y_1 = Y_{-1}$ such that (a) $Y_1 - Y_{e,1} = 0$ and (b) $\pi^T < \pi_1 = \pi_0$; furthermore, (c) $i_1^{MP} = i_0^{MP}$.

- The new constant inflation rate configuration ($t = 2$ and thereafter)

  As consequence of last-period no interest-rate change (and no new p_o variation either), in $t = 2$ aggregate demand is unchanged, that is, $Y_2 = Y_1$, the same as equilibrium output, that is, $Y_{e,2} = Y_{e,1}$. Consequently, the inflation rate will be:

$$\begin{aligned}\pi_2 &= \pi^T + A + \alpha(Y_2 - Y_{e,2})\\ &= \pi^T + A + \alpha(0)\\ &= \pi^T + A\\ &= \pi_1\end{aligned}$$

that is, the inflation rate will remain the same as well. In turn, given the CB does not intervene, then $i_1^{MP} = i_2^{MP}$ will be the case.

Summarizing, the economy ends up $t = 2$ with $Y_2 = Y_1$ and $Y_{e,2} = Y_{e,1}$ such that (a) $Y_2 - Y_{e,2} = 0$ and (b) $\pi^T < \pi_2 = \pi_1$; furthermore (c) $i_2^{MP} = i_1^{MP}$.

The economy has reached a new configuration with a constant inflation rate, even if it is permanently higher than the CB's target. In this new configuration: (i) the inflation rate is constant and permanently above the target, i.e., $\pi_k = \pi^T + \alpha A$ for $0 \leq k$; (ii) the stabilizing interest rate is the same as in the original MRE, i.e., $i_{-1}^{MP} = i_k^{MP}$ for $0 \leq k$; (iii) actual and equilibrium output are the same as in the original MRE, i.e., $Y_{-1} = Y_k = Y_{e,k} = Y_{e,-1}$ for $1 \leq k$.

ii. The fact that the shock is temporary makes that, even if the CB does not intervene, the shock (a) reduces equilibrium output only during one period

$(Y_{e,0} < Y_{e,-1}$ but later $Y_{e,-1} = Y_{e,1} = Y_{e,2} = ...)$, (b) does not create a recession $(Y_{-1} = Y_0 = Y_1 = Y_2 = ...)$, and (c) increases the inflation rate once but then it remains constant in that higher level (for every $0 \leq k$ period, $\pi_k = \pi^T + A$ is the case). This is certainly different to the permanent shock case, in which, if the CB did not intervene, the reduction in equilibrium output is permanent and the inflation rate increases every period (so that, recall, after k periods, $\pi_k = \pi^T + (k+1)A$).

iii. ◦ The shock's period $(t = 0)$

To begin with, the reduction in p_o reduces equilibrium output, that is, $Y_{e,0} < Y_{e,-1}$. Furthermore, we assumed that it leaves equilibrium demand unchanged, that is, $Y\_1 = Y_0$. Under this assumption, we can conclusively state that a positive output gap arises, that is, $0 < A = \alpha(Y_0 - Y_{e,0})$. This positive output gap will increase the inflation rate above the CB's target during the shock's period. Specifically, from Equation (60) it follows that:

$$\begin{aligned} \pi_0 &= \pi^T + \alpha(Y_0 - Y_{e,0}) \\ &= \pi^T + A \\ &> \pi^T \end{aligned}$$

But, then, as response, the CB will carry out contractionary monetary policy, which in this period means it'll increase the interest rate. i.e., set $i_{-1}^{MP} < i_0^{MP}$. However, given its forward-looking and highly competent, the CB will increase it rightly forecasting (since this $t = 0$) (a) that the shock is temporary, so that, if it did not intervene, then nevertheless the output gap would close; which implies it accurately predicts next period's PC ($\pi_1 = \pi^T + \alpha A + \alpha(Y_1 - Y_{e,1})$ but also having clear that its $i_{-1}^{MP} < i_0^{MP}$ decision will increase equilibrium output beyond its original value); and (b) the exact amount in which i_0^{MP} has to be above i_{-1}^{MP} to achieve that $Y_1 - Y_{e,1} = -A < 0$; which implies it accurately understands that (given that $p_{o,1} = p_{o,-1}$ again, which by itself would close the gap) the positive difference $0 < i_0^{MP} - i_{-1}^{MP}$ needs to be smaller than if the shock were permanent, that is, the CB accurately understands it does not need to increase the interest-rate as much as it would have to in the scenario of a permanent shock.

Summarizing, the economy ends up $t = 0$ with $Y_{e,0} < Y_{e,-1}$ and $Y_{-1} = Y_0$ so that (a) $0 < A = Y_0 - Y_{e,0}$, (b) $\pi^T < \pi_0$, and (c) $i_{-1}^{MP} < i_0^{MP}$.

- Following to the shock's period ($t = 1$)

  As consequence of the last-period's interest-rate rise, in $t = 1$ aggregate demand decreases, i.e., $Y_1 < Y_0$, and equilibrium output increases, i.e., $Y_{e,0} < Y_{e,1}$. Furthermore (because of the high rationality and capabilities of the CB) these effects are such that they create a negative output gap which is equal, in absolute value, to the last-period's positive gap, i.e., $Y_1 - Y_{e,1} = -\gamma A < 0$; but don't overlook that, because the return of the oil price to its original, higher value, already increases equilibrium output (so that $Y_{e,0} < Y_{e,1}$ even in the absence of the monetary policy tightening), this negative gap implies an smaller reduction of effective output that if the oil price reduction were permanent, that is, the difference $Y_1 - Y_0 < 0$ if the shock is temporary is smaller than the analogous difference if the shock were permanent. In any case, consequently, the inflation rate will be:

$$\begin{aligned}\pi_1 &= \pi^T + \alpha A + \alpha(Y_1 - Y_{e,1}) \\ &= \pi^T + \alpha A - \alpha A \\ &= \pi^T,\end{aligned}$$

that is, the inflation rate aligns with the CB's target. Having achieved this outcome, then, in $t = 1$ the CB will reduce the interest rate, i.e., set $i_1^{MP} < i_0^{MP}$. However, given its forward-looking and highly competent, the CB will reduce it rightly forecasting (since $t = 1$) (a) that next period's PC will be given that $\pi_2 = \pi^T + \alpha(Y_2 - Y_{e,2})$, and (b) the exact amount in which i_1^{MP} has to be below i_0^{MP} to achieve that $Y_2 - Y_{e,2} = 0$. Furthermore, given it accurately understands that the shock is temporary and it left equilibrium aggregate demand overall unchanged, (c) it accurately understands that the i_1^{MP} needs to be the same as the, original MRE's i_{-1}^{MP}.

Summarizing, the economy ends up $t = 1$ with $Y_1 < Y_0$ and $Y_{e,0} < Y_{e,1}$ such that (a) $Y_1 - Y_{e,1} = -A < 0$, (b) $\pi_1 = \pi^T$, and (c) $i_{-1}^{MP} = i_1^{MP} < i_0^{MP}$.

- The new MRE ($t = 2$ and thereafter)

  As consequence of last-period's interest-rate reduction, in $t = 2$ aggregate demand is boosted, i.e., $Y_1 < Y_2$, and equilibrium output reduced, i.e., $Y_{e,2} < Y_{e,1}$. Furthermore (because of the high rationality and capabilities of the CB) these effects are such that they exactly close the output gap, i.e., $Y_2 - Y_{e,2} = 0$. Consequently, the inflation rate will be:

$$\begin{aligned}\pi_2 &= \pi^T + \alpha(Y_2 - Y_{e,2}) \\ &= \pi^T + \alpha(0) \\ &= \pi^T,\end{aligned}$$

  that is, it'll remain at the CB's target; furthermore, now it's constant as well: $\pi^T = \pi_2 = \pi_0$. Having achieved this outcome, the CB will set the same interest rate, i.e., it'll set $i_2^{MP} = i_1^{MP}$. More in detail, because of its high rationality and capabilities, the CB rightly forecast (since $t = 2$) that, if it keeps the same interest-rate, next period's PC will be the same as this period's and, also, the output gap will remain zero. Furthermore, given the shock is temporary and left aggregate demand overall unchanged, it accurately understands that it needs to be the same as the original MRE's interest-rate.

Summarizing, the economy ends up $t = 2$ with $Y_1 < Y_2$ and $Y_{e,2} < Y_{e,1}$ such that (a) $Y_2 - Y_{e,2} = 0$, (b) $\pi_2 = \pi^T$, and (c) $i_2^{MP} = i_1^{MP}$.

The economy has reached a new MRE configuration. It's such that the economy is back to exactly the same original MRE configuration: (i) the inflation rate is constant and equal to the CB's target, i.e., $\pi^T = \pi_k = \pi_{k+1}$ for $1 \leq k$; (ii) the stabilizing interest rate is the same, i.e., $i_{-1}^{MP} = i_k^{MP}$ for $1 \leq k$; (iii) actual and equilibrium output are the same, i.e., $Y_{-1} = Y_{e,-1} = Y_{e,k} = Y_k$ for $1 \leq k$.

iv. The main takeaway of the temporary oil price reduction analysis is the following: it shows that the economy experiences a softer cycle, that is, the reduction in effective output needed to undo the inflationary inertia is smaller than in the scenario in which the oil price reduction is permanent.

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