COURSES

C1. **Diego Córdoba**, Instituto de Ciencias Matemáticas, España

Active scalars with singular incompressible velocities

The goal of these lectures is to present the main ideas and arguments of recent results concerning global solutions and finite time singularities for a family of incompressible fluids. In particular, we study the Surface Quasi-Geostrophic equation (SQG) and the Incompressible Porous Media equation (IPM). SQG and IPM are the two simplest possible physical models that capture the conservative quantities and the non-local structure of the incompressible 3D Euler equations.

C2. **Carlos Matheus**, Instituto Nacional de Matematica Pura e Aplicada IMPA, Brasil

The fine structure of the Lagrange and Markov spectra

The Lagrange and Markov spectra are two subsets L and M of the real line related to classical Diophantine approximation problems in Number Theory.

The structure of L and M attracted the interest of several mathematicians:

a) Hurwitz showed in 1891 that $\sqrt{5}$ is the smallest number in L;
b) Markov showed in 1879 that $L \cap (-\infty, 3) = M \cap (-\infty, 3)$ consists of an explicit increasing sequence k_n of quadratic surds converging to 3 (e.g., $k_1 = \sqrt{5} < k_2 = 2\sqrt{2} < k_3 = \sqrt{221}/5 < \ldots$);
b) Hall proved in 1947 that L and M contain an infinite half-line $[c, +\infty)$ and Freiman determined in 1975 the biggest half-line contained in L, namely $[c_F, +\infty)$ where $c_F = \frac{2221564096+283748\sqrt{462}}{491993569} \simeq 4.52782956...$

The goal of our course is to explain how the interplay between L, M and the dynamics of the continued fraction algorithm led C. G. Moreira to prove that the transition from the discrete set $L \cap (-\infty, 3) = M \cap (-\infty, 3)$ to the half-line $L \cap [c_F, +\infty) = M \cap [c_F, +\infty) = [c_F, +\infty)$ occurs in a highly non-trivial way: for example, the Hausdorff dimension of $L \cap (-\infty, t)$ varies continuously with the parameter t.

C3. **Ulrike Tillmann**, Oxford University, Inglaterra

Topological quantum field theories in homotopy theory

Topological quantum field theories (TQFT) were axiomatized by Atiyah and Segal in the late 1980s. Motivated by physics, they provided a new framework in which invariants of manifolds arise naturally. From this axiomatic point of view, a TQFT is a functor from a category of closed manifolds and cobordisms to a suitable category of vector spaces.

Prior to this, cobordism was also an important concept for the classification of manifolds in the work of Thom and others in the 1950s. There seemed however to be no relation between the two appearances of cobordisms.

In this lecture I will explain how a homotopy theoretic approach to TQFTs brought these together and how recent results shed light on both, TQFTs and classical cobordism theory, and has contributed to our understanding of manifolds.

I will explain what TQFTs are, recall the classical theory of Thom, discuss the work of Lurie on the Baez-Dolan cobordism hypothesis and interpret a refined version of the theorem of Galatius-Madsen-Tillmann-Weiss in this context. Time permitting, we will relate this to the Madsen-Weiss theorem solving the Mumford conjecture and the recent work of Galatius and Randal-Williams on a higher dimensional analogue of that

C4. **Alberto Verjovsky**, Universidad Nacional Autónoma de México UNAM, México

Intersección de cuádricas en $\mathbb{C}^n$, variedades ángulo-momento, variedades complejas y tóricas y politopos convexos

El curso trata sobre la construcción y estudio de estructuras geométricas de variedades *ángulo-momento.* Estas son variedades que admiten la acción de un toro real $\mathbb{T}^n = \mathbb{S}^1 \times \cdots \times \mathbb{S}^1$ de tal suerte que el espacio de órbitas es un politopo convexo simple [3]. Las variedades que se describirán y estudiarán principalmente son las las llamadas *variedades LV-M* o *LVMB* ([1], [2], [4], [5], [6]). Estas variedades compactas de dimensión impar M^{2n+1} se obtienen como intersección de hipersuperficies cuadráticas en posición general en $\mathbb{C}^{n+2}$ y la esfera $\mathbb{S}^{2n+3}$. Las variedades admiten una acción localmente libre del círculo de tal suerte que el cociente es una variedad compleja N^n ($dim_{\mathbb{C}} N = n$) que en general no es de tipo Kähler. La variedad N admite una fibración holomorfa $\pi : N \to V$ de tipo Seifert sobre una variedad tórica V y con fibra un toro compacto complejo. Las posibles singularidades de V son de tipo *orbifold* (singularidades simples). Toda variedad compacta tórica con singularidades simples se obtiene por este proceso. Las variedades ángulo momento M^{2n+1} admiten la acción del toro $\mathbb{T}^{n+2}$ cuyo cociente es un politopo convexo K^{n-1}. En analogía con las variedades tóricas la combinatoria de este politopo convexo controla la geometría y topología de M^{2n+1} y N^{2n}. Las variedades M^{2n+1} admiten una estructura de contacto. También admiten una estructura de *libro abierto* que tiene como páginas variedades complejas. Existen muchos problemas abiertos interesantes.

References

[1] F. Bosio. "Variétés complexes compactes : une généralisation de la construction de Meersseman et López de Medrano-Verjovsky". *Ann. Inst. Fourier.* 51, 2001, 1259–1297

[2] F. Bosio, L. Meersseman. "Real quadrics in $\mathbb{C}^n$, complex manifolds and convex polytopes". *Acta Math.* **197**, 2006, 53–127

[3] V.M. Buchstaber, T.E. Panov. Torus actions and their applications in topology and combinatorics. *Publications AMS* Providence 2002

[4] S. López de Medrano, A. Verjovsky. "A new family of complex, compact, non symplectic manifolds". *Bol. Soc. Mat. Bra.* 28, # 2, 1997, 253–269

[5] L. Meersseman. "A new geometric construction of compact complex manifolds in any dimension" *Math. Ann.* 317, 2000, 79–115

[6] "Holomorphic principal bundles over toric projective varieties" *J. Reine Angew. Math.* 572, 2004, 57–96