# INVITED TALKS

# IT1. Federico Ardila, San Francisco State University, USA

## Moving robots efficiently using the combinatorics of CAT(0) cubical complexes

In this talk we will use tools from geometric group theory and combinatorics to plan the motion of a robot. Given a reconfigurable system X, such as a robot moving on a grid or a set of particles moving around a graph without colliding, the moduli space of all possible positions of X is a cubical complex S(X). When S(X) is non-positively curved (CAT(0)), we can explicitly construct the shortest path between any two points.

We show that CAT(0) cubical complexes are in bijection with posets with inconsistent pairs (PIPs); therefore we can prove that a state complex S(X) is CAT(0) by identifying the corresponding PIP. In applications, the PIP serves as a combinatorial "remote control" to move robots efficiently from one position to another. We use this very general framework to solve the problem of efficiently moving a robotic arm in a tunnel. Along the way we encounter lots of interesting combinatorics.

This talk is based on a series of papers with Tia Baker, Hanner Bastidas, Cesar Ceballos, John Guo, Megan Owen, Seth Sullivant, and Rika Yatchak. It will assume no previous knowledge of the subject.

IT2. Mikhail Belolipetsky, Instituto Nacional de Matematica Pura e Aplicada IMPA, Brasil

# Arithmetic hyperbolic reflection groups

A hyperbolic reflection group is a discrete group generated by reflections in the faces of an n-dimensional hyperbolic polyhedron. The study of higher dimensional hyperbolic reflection groups has a long and remarkable history which goes back to the pioneering papers of Makarov and Vinberg. In the recent years there has been a new wave of activity in this area which led to the solution of an important open problem about finiteness of the number of commensurability classes of arithmetic hyperbolic reflection groups. On the talk I will review some of the recent results and discuss some other related open problems.

## IT3. María Emilia Caballero, Universidad Nacional Autónoma de México UNAM, México

# Stable Lévy processes, Lamperti's representations and generalizations

The relations discovered by Lamperti (1967, 1972) between Lévy processes (LP) and other interesting Markov processes have been widely studied, in part because he announced some interesting results without proof and also because all the possibilities they offer. In the past ten years, and continuing the work done by serval authors (Jirina, Grimwall, Helland, Ethier, Kurtz, Carmona, Petit, Yor, Bertoin), two of these relations have been widely exploited, and this has been very fruitful on several directions:

- Construction of a great range of useful processes, some of which are very good models in different applications.
- An answer to the question raised by Lamperti on the entrance law of Positive Self Similar Markov Processes (PSSMP).
- The use of known facts of LP to obtain properties of their transformed processes (asymptotic results, overshoots, exit laws....) and reciprocally.
- Enrich the cases where the Wiener-Hopf factorization can be explicitly given.
- Generalizations of the Lamperti transformation.
- Characterization of Continuous State Branching Processes (CSBP), with or without immigration, and affine processes.
- Methods via Stochastic Differential Equations (SDE).

In this talk I shall give a panorama of these surprising developments, focusing first on the brownian case and then on the stable case. This will allow us to give the main ideas behind the wide range of new results and grasp the main methods, without treating the general case of all Lévy processes, which would be a task beyond the possibilities of a 50 minutes talk. We will also explain, how the SDE enter in this picture, in part inspired by Doeblin's ideas (re-)discovered in 2000.

Until 2014-15 all known results were for the one dimensional case, due to the nature of the Lamperti's transformations. The main issue is that in recent months [7] and [10] a breakthrough has been achieved in this line of research by constructing them in several dimensions. In both cases the generalization is not trivial and offers a whole new perspective on this subject.

Some references appear here, but the list is much longer, this is just an example of the different lines of development that are in course. (\*\*The slides will be in spanish and the talk in english).

## References

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#### IT4. Conchita Martínez, Universidad de Zaragoza, Spain

## Recent results on the proper dimension of certain groups

The proper dimension of an arbitrary group is the smallest possible dimension of a classifying space for proper actions. In this talk we will review the relevance of this notion and some recent results that allow us to determine this invariant for certain well known families of groups. Parts of this talk correspond to joint work with Javier Aramayona, Dieter Degrijse, Peter Kropholler, Brita Nucinkis and Juan Souto.

IT5. Ursula Molter, Universidad de Buenos Aires and IMAS-CONICET, Argentina

#### The Amalgan Balian Low Theorem and time-frequency shift invariance

The Balian-Low Theorem expresses the fact that time-frequency concentration and non-redundancy are essentially incompatible. Specifically, if  $\varphi \in L^2(\mathbb{R})$ ,  $\Lambda \subset \mathbb{R}^2$  is a lattice and the system  $(\varphi, \Lambda) = \{e^{2\pi i \eta x} \varphi(x-u) : (u, \eta) \in \Lambda\}$  is a Riesz basis for  $L^2(\mathbb{R})$ , then  $\varphi$  satisfies

$$\left(\int (x-a)^2 |\varphi(x)|^2 \, dx\right) \cdot \left(\int (\omega-b)^2 |\widehat{\varphi}(\omega)|^2 \, d\omega\right) = \infty, \quad a, b \in \mathbb{R}.$$

The Amalgam Balian-Low Theorem states that if  $(\varphi, \alpha \mathbb{Z} \times \beta \mathbb{Z})$  is a Riesz basis for  $L^2(\mathbb{R})$ , then  $\varphi$  cannot belong to the Feichtinger algebra  $S_0(\mathbb{R})$ , a class of functions decaying well in time and frequency. Precisely,

$$S_0(\mathbb{R}) = \left\{ f \in L^2(\mathbb{R}) : Vf(t,\nu) = \int f(x)e^{-(x-t)^2} e^{2\pi i x\nu} \, dx \in L^1(t,\nu) \right\}.$$

Note that  $Vf(t,\nu) \in L^2(t,\nu) \cap L^{\infty}(t,\nu)$  for all  $f \in L^2(\mathbb{R})$  and the requirement  $Vf(t,\nu) \in L^1(t,\nu)$  essentially necessitates  $L^1$  decay of f and of its Fourier transform  $\hat{f}$ . This space is called the *Feichtinger algebra*.

Let  $T_u f(x) = f(x - u)$ , and  $M_\eta f(x) = e^{2\pi i \eta x} f(x)$ , denote the usual translation and modulation operators, and let  $\pi(u, \eta) = M_\eta T_u$ , (with  $u \in \mathbb{R}$  and  $\eta \in \widehat{\mathbb{R}}$  the dual group of  $\mathbb{R}$ ) denote the time-frequency shift. For  $\varphi \in L^2(\mathbb{R})$  and a lattice  $\Lambda \subset \mathbb{R} \times \widehat{\mathbb{R}}$ , let  $\mathcal{G}(\varphi, \Lambda)$  denote the *Gabor spaces*,  $\mathcal{G}(\varphi, \Lambda) := \overline{\operatorname{span}\{\pi(\lambda)\varphi\}}$ , where  $\overline{V}$  is the closure of V in  $L^2(\mathbb{R})$ .

In this talk we address the question whether there may exist a  $\mu \in \mathbb{R} \times \widehat{\mathbb{R}} \setminus \Lambda$  with  $\pi(\mu)\varphi \in \mathcal{G}(\varphi, \Lambda)$ . The result relates the existence of such  $\mu$ , to the fact that  $\varphi$  belongs (or does not belong) to the *smoothness space*  $S_0(\mathbb{R})$ . We have

**Theorem.** If  $(\varphi, \Lambda)$  is a Riesz basis for its closed linear span  $\mathcal{G}(\varphi, \Lambda)$  with  $\varphi \in S_0(\mathbb{R})$  and the density of the lattice  $\Lambda$  is rational, then for any  $(u, \eta) \notin \Lambda \pi(u, \eta) \varphi \notin \mathcal{G}(\varphi, \Lambda)$ .

Note that  $(\varphi, \Lambda)$  being a Riesz basis for  $L^2(\mathbb{R})$  implies that the density of  $\Lambda$  equals  $1 \in \mathbb{Q}$ ; and  $\mathcal{G}(\varphi, \Lambda) = L^2(\mathbb{R})$  implies that  $\pi(u, \eta)\varphi \in \mathcal{G}(\varphi, \Lambda)$  for all  $(u, \eta) \in \mathbb{R} \times \widehat{\mathbb{R}}$ . Therefore the theorem implies that  $\varphi \notin S_0(\mathbb{R})$ . Joint work with:

- Carlos Cabrelli, University of Buenos Aires and IMAS-CONICET, Argentina
- Götz Pfander, Philipps-Universität Marburg, Germany
- Dae Gwan Lee, Philipps-Universität Marburg, Germany
- IT6. Andrés Navas, Universidad de Santiago de Chile, Chile

## On the large-scale geometry of tilings

We consider tilings of the plane by polygons, and we discuss the question of whether this is equivalent, in the Lipschitz sense, to the standard tiling by unit squares. As an example, we will show that this is the case for the famous Penrose tiling, but not for "most" tilings.

# IT7. Rafael Potrie, Universidad de la República, Uruguay

# Dynamics in the study of discrete subgroups of Lie groups

We are interested in two problems regarding representations of discrete groups in Lie groups. The first one is related with the description of open subsets of faithful representations whose image is discrete. The second one is related with understanding conditions under which one can ensure that the image of the representation is contained in a proper Lie subgroup. We shall explain some results in those directions, in particular, how to use dominated splittings to obtain open sets of quasi-isometric representations and a result regarding entropy rigidity for Hitchin representations. Both problems will be looked from a dynamical point of view and are related with Labourie's notion of Anosov representations. We hope to make evident the relation with the theory of linear cocycles and thermodynamical formalism for Anosov flows and pose natural questions which arise from this point of view.

IT8. Noemi Wolanski, Universidad de Buenos Aires, Argentina

# Asymptotic behavior of solutions to a nonlocal diffusion equation on manifolds

I will present joint work with C. Bandle, M. del Mar Gonzalez and M. Fontelos on different asymptotic questions for a nonlocal diffusion problem determined by a smooth kernel of compact support. The main questions are if and how does the geometry of the manifold influence different kinds of asymptotic problems such as the infinitesimal limit as the support of the kernel contracts to 0 or the large time asymptotic behavior of the solution for a fixed kernel.