PLENARY TALKS

PT1. **Fernando Codá Marques**, Princeton University, USA

Morse index and multiplicity of min-max minimal hypersurfaces

The Min-max Theory for the area functional, started by Almgren in the early 1960s and greatly improved by Pitts in 1981, was left incomplete because it gave no Morse index estimate for the min-max minimal hypersurface. Nothing was said also about the fundamental problem of multiplicity. In this talk I will describe our current efforts to develop the theory further. I will discuss the first general Morse index bounds for minimal hypersurfaces produced by the theory. We also settle the multiplicity problem for the classical case of one-parameter sweepouts. If time permits I will mention some conjectures for the field. This is based on joint work with Andre Neves.

PT2. **Ruy Exel**, Universidade Federal de Santa Catarina, Brazil

Partial actions and the Banach-Tarski paradox

In 1924, Banach and Tarski showed that a solid ball in $\mathbb{R}^3$ can be broken into finitely many pieces which, in turn, can be used as a puzzle to form two balls identical to the original one. This result is so striking that it bacame known as the *Banach-Tarski paradox*, despite being a theorem, proven with full mathematical rigor.

The Banach-Tarski paradox has important consequences in Mathematics, one of which is the impossibility of defining a notion of "volume", applicable to any subset of $\mathbb{R}^3$. Otherwise the volume of the initial ball, which is $\frac{4}{3}\pi R^3$, would coincide with the sum of the volumes of the pieces in our puzzle, which in turn is $2 \times \frac{4}{3}\pi R^3$.

Tarski later found a reciprocal of this *paradox*, proving that, under the action of a group of symmetries, the only obstruction to the existence of an invariant measure is the presence of *paradoxical* sets, namely those which can be doubled in size, as the ball in $\mathbb{R}^3$

In this lecture I plan to take the ideas of Banach and Tarski into the realm of totally disconnected topological spaces, by restricting the notion of measure to the boolean algebra of clopen (closed and open) sets. We will see that the theory of partial group actions, with which I have been involved for the past 25 years, provides an example forbidding the generalization of Tarski's Theorem to this context. This example was found in collaboration with Pere Ara, from the Universitat Autónoma de Barcelona.

PT3. **Pablo Augusto Ferrari**, Universidad de Buenos Aires, Argentina

Fan of characteristics and TASEP hydrodynamics

The totally asymmetric simple exclusion process, TASEP, is a stochastic model of particles jumping in the integers subject to a exclusion rule. Rescaling space and time, the density of particles converges to the solution of the Burgers equation, this is called hydrodynamic limit. We couple the TASEP starting with ordered initial configurations, one for each density R in [0,1] and a second class particle for each of these configurations. We show that the rescaled positions of those particles converge to a fan which coincides with the rarefaction fan of characteristics of the Burgers equation starting with a Heaviside configuration. We use this fact to prove the hydrodynamic limit without using subadditivity.

PT4. **Mikhail Lyubich**, Stony Brook University, USA

Renormalization and Area of Julia sets

Renormalization is a central idea of contemporary Dynamical Systems Theory. It allows one to control small scale structure of certain classes of systems, which leads to universal features of the phase and parameter spaces. We will review several occurancies of Renormalization in Holomorphic Dynamics (especially, for quadratic-like and Siegel maps) that enlighten the structure of many Julia sets and the Mandelbrot set. In particular, these ideas helped to construct examples of Julia sets of positive area (resolving a classical problem in this field).

First examples were constructed by Buff and Cheritat about 10 years ago, and more recently a different class, with many new features, was produced by Artur Avila and the author. In the talk, we will describe these developments.

PT5. **Gunther Uhlmann**, University of Washington, USA

Journey to the Center of the Earth

We will consider the inverse problem of determining the sound speed or index of refraction of a medium by measuring the travel times of waves going through the medium. This problem arises in global seismology in an attempt to determine the inner structure of the Earth by measuring travel times of earthquakes. It has also several applications in optics and medical imaging among others.

The problem can be recast as a geometric problem: Can one determine a Riemannian metric of a Riemannian manifold with boundary by measuring the distance function between boundary points? This is the boundary rigidity problem. We will also consider the problem of determining the metric from the scattering relation, the so-called lens rigidity problem. The linearization of these problems involve the integration of a tensor along geodesics, similar to the X-ray transform.

We will also describe some recent results, join with Plamen Stefanov and Andras Vasy, on the partial data case, where you are making measurements on a subset of the boundary. No previous knowledge of Riemannian geometry will be assumed.

PT6. **María Ronco**, Universidad de Talca, Chile

Algebraic operads and combinatorial Hopf algebras

We want to motivate the study of underlying algebraic structures on combinatorial Hopf algebras.

The subspace of primitive elements of any bialgebra has a natural structure of Lie algebra. If the base field has characteristic zero and the bialgebra H is cocommutative and conilpotent, the Cartier-Milnor-Moore Theorem states that the Lie algebra of primitive elements of H determines the whole bialgebra structure of H. When H is not cocommutative, the Lie bracket does not suffice to reconstruct H, but in some cases there exist finer structure on the subspace of primitive elements which completely determine the bialgebra structure.

Probably the most well-known example of non cocommutative bialgebras is the Connes-Kreimer Hopf algebra, appearing in the context of renormalisation theory. Its subspace of primitive elements is the free pre-Lie algebra on one element.

Higher non-commutative versions of this bialgebra, led us to introduce algebraic theories (operads) satisfying that:

1. the algebras over these operads are associative algebras equipped with some additional products,
2. there exist in all cases a natural notion of bialgebra,
3. the Lie bracket on the subspace of primitive elements comes from finer structures like multibrace algebras.

There exist in literature many examples of these types of structures coming from physics, as the Faá di Bruno bialgebra and the Baxter algebras, as well as from algebraic topology, as the cacti algebras and the Gerstenhaber algebras.

In all the examples, combinatorics provide an important tool in the study of free objects for these new algebraic theories, as well as in the understanding of the relations between them. In the last years, an important amount of work has been done on generalisations of associahedra, on one hand, and on combinatorial descriptions of operads related to higher structures on cohomology on the other side. Our goal is to give a brief account of the state of art on the subject.