Parcial II

① $H(r,t)=3re^{2t}$

a) $\nabla H(r,t)=\langle 3e^{2t}, 6re^{2t}\rangle$

$$\boxed{\nabla H(\tfrac{7}{3},0)=\langle 3, 14\rangle}$$

b) $\vec{PQ}=\langle \frac{1}{3}-\frac{7}{3}, 3-0\rangle=\langle -2,3\rangle$

$\|\langle -2,3\rangle\|=\sqrt{4+9}=\sqrt{13}$

$\Rightarrow \vec{u}=\frac{1}{\sqrt{13}}\langle -2,3\rangle$

$D_{\vec{u}}H(\frac{7}{3},0)=\langle 3,14\rangle\cdot\left(\frac{1}{\sqrt{13}}\langle -2,3\rangle\right)=\frac{1}{\sqrt{13}}(-6+42)$

$=\frac{36}{\sqrt{13}}$.

② $f(x,y)=x^3-y^2-12x-2y$

$\Rightarrow f_x=3x^2-12=3(x^2-4)=0 \Rightarrow x=-2$ ó $x=2$

$f_y=-2y-2=-2(y+1)=0 \Rightarrow y=-1$.

Puntos críticos $(-2,-1)$ y $(2,-1)$.

$f_{xy}=0$, $f_{xx}=6x$, $f_{yy}=-2$

$\Rightarrow d(x,y)=-12x-0=-12x$

Luego,

* $d(-2,-1)=24>0$ y $f_{xx}(-2,-1)=-12<0$
  $\Rightarrow (-2,-1)$ es un máximo.
* $d(2,-1)=-24<0 \Rightarrow (2,-1)$ es un punto de silla.

3 $f(x,y,z) = xyz + 5$ restricción $x^3+y^3+z^3 = 24$.

$$\begin{cases} yz = 3\lambda x^2 \to \frac{yz}{x^2} = 3\lambda \quad ① \\ xz = 3\lambda y^2 \to \frac{xz}{y^2} = 3\lambda \quad ② \\ xy = 3\lambda z^2 \to \frac{xy}{z^2} = 3\lambda \quad ③ \\ x^3+y^3+z^3 = 24 \end{cases}$$

$g(x,y,z) = x^3+y^3+z^3$

$\nabla f = \langle yz, xz, xy \rangle$

$\nabla g = \langle 3x^2, 3y^2, 3z^2 \rangle$

Al igualar ① y ②

$$\frac{yz}{x^2} = \frac{xz}{y^2} \Rightarrow y^3 = x^3 \Rightarrow y = x$$

Al igualar ① y ③

$$\frac{yz}{x^2} = \frac{xy}{z^2} \Rightarrow z^3 = x^3 \Rightarrow z = x.$$

Luego,

$$x^3 + x^3 + x^3 = 24 \Rightarrow x^3 = 8 \Rightarrow \boxed{x=2}.$$

Ahora, notemos que $(\sqrt[3]{24}, 0, 0)$ cumple la restricción

$(\sqrt[3]{24})^3 = 24$.

• $f(2,2,2) = 13$ y $f(\sqrt[3]{24}, 0, 0) = 5$, por lo tanto, $(2,2,2)$ es un máximo.

4
$$\begin{cases} 2x = 2\lambda(x-1) \to x = \frac{-\lambda}{1-\lambda} \\ 2y = 2\lambda(y-3) \to y = \frac{-3\lambda}{1-\lambda} = 3x \\ 2z = 2\lambda(z-5) \to z = -5\lambda/(1-\lambda) = 5x \\ (x-1)^2 + (y-3)^2 + (z-5)^2 = 560 \end{cases}$$

$$\Rightarrow (x-1)^2 + (3x-3)^2 + (5x-5)^2 = 560 \Rightarrow 16(x-1)^2 = 560$$

$$\Rightarrow (x-1)^2 = 16 \Rightarrow x = 5 \text{ ó } x = -3$$

$P(5,15,25) = 875 \to (5,15,25)$ Máx y $P(-3,-9,-15) = 315 \to (-3,-9,-15)$ Mín