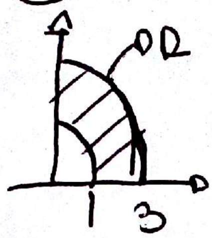
① **Parcial III**



R: $0 \leq \theta \leq \frac{\pi}{2}$, $\quad 1 \leq r \leq 3$

$\rho(x,y) = 3x \Rightarrow \rho(r,\theta) = 3r\cos\theta$

$$m = \iint_R \rho(x,y)\,dA = \int_0^{\frac{\pi}{2}}\int_1^3 \underbrace{3r\cos\theta \cdot r}_{3r^2\cos\theta}\,dr\,d\theta = \int_0^{\frac{\pi}{2}} \frac{\cancel{3}r^3}{\cancel{3}}\cos\theta \Big|_1^3 d\theta$$

$$= (27-1)\int_0^{\frac{\pi}{2}} \cos\theta\,d\theta = 26\,\mathrm{sen}\,\theta\Big|_0^{\frac{\pi}{2}} = \boxed{26}$$

$$M_y = \iint_R x\,\rho(x,y)\,dA = \int_0^{\frac{\pi}{2}}\int_1^3 \underbrace{(r\cos\theta)(3r\cos\theta)\,r}_{3r^3\cos^2\theta}\,dr\,d\theta$$

$$= 3\int_0^{\frac{\pi}{2}} \frac{r^4}{4}\cos^2\theta\Big|_1^3 d\theta = \frac{3}{4}(3^4-1)\int_0^{\frac{\pi}{2}} \frac{\cos(2\theta)+1}{2}\,d\theta$$

$$= \frac{60}{2}\int_0^{\frac{\pi}{2}} (\cos(2\theta)+1)\,d\theta = 30\left(\frac{1}{2}\mathrm{sen}(2\theta) + \theta\right)\Big|_0^{\frac{\pi}{2}}$$

$$= 30\,\frac{\pi}{2} = \boxed{15\pi}$$

$$M_x = \iint_R y\,\rho(x,y)\,dA = \int_0^{\frac{\pi}{2}}\int_1^3 \underbrace{(r\,\mathrm{sen}\,\theta)(3r\cos\theta)\,r}_{3r^3\,\mathrm{sen}\,\theta\cos\theta}\,dr\,d\theta$$

$$= 3\int_0^{\frac{\pi}{2}} \frac{r^4}{4}\,\mathrm{sen}\,\theta\cos\theta\Big|_1^3 d\theta = \frac{3}{4}(3^4-1)\cdot\frac{1}{2}\,\mathrm{sen}^2\theta\Big|_0^{\frac{\pi}{2}}$$

$$= \frac{3}{4}\left(\frac{80}{2}\right)\left(\mathrm{sen}^2\frac{\pi}{2} - \mathrm{sen}^2 0\right) = \boxed{30}$$

Centro masas $\left(\frac{30}{26}, \frac{15\pi}{26}\right)$.

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(2) $A(S) = \iint_R \sqrt{1+[f_x]^2+[f_y]^2}\, dA$

R
$0 \leq r \leq 3$
$0 \leq \theta \leq 2\pi$

$f_x = -2x \quad , \quad f_y = -2y$

$$A(S) = \int_0^{2\pi}\int_0^{3} \sqrt{\underbrace{1+4r^2\cos^2\theta + 4r^2 \operatorname{sen}^2\theta}_{1+4r^2}}\; r\,dr\,d\theta = 4\int_0^{\frac{\pi}{2}}\int_0^{3} \sqrt{1+4r^2}\; r\,dr\,d\theta$$

$$= 4\int_0^{\frac{\pi}{2}} \frac{2}{3}\cdot\frac{1}{8}(1+4r^2)^{3/2}\Big|_0^3 d\theta = \frac{1}{3}\int_0^{\pi/2} (1+4\cdot 3^2)^{3/2} - 1\, d\theta$$

$$= \frac{1}{3}\cdot\frac{\pi}{2}(37^{3/2}-1) = \frac{\pi}{6}(37^{3/2}-1).$$

(3) $$\int_0^1\int_0^1\int_0^{2-x^2-y^2} xye^z\, dz\,dx\,dy = \int_0^1\int_0^1 (xye^{2-x^2-y^2} - xy)\,dx\,dy$$

$$= \int_0^1 \left(-\frac{1}{2}ye^{2-x^2-y^2} - \frac{xy^2}{2}\right)\Big|_0^1 dy = \int_0^1 -\frac{1}{2}ye^{1-y^2} + \frac{1}{2}ye^{2-y^2} - \frac{y}{2}\, dy$$

$$= \left(\frac{1}{2}\cdot\frac{1}{2}e^{1-y^2} - \frac{1}{2}\cdot\frac{1}{2}e^{2-y^2} - \frac{y^2}{4}\right)\Big|_0^1 = \frac{1}{4}(\cancel{e^0} - e - \cancel{1} - e + e^2 + 0)$$

$$= \frac{1}{4}(e^2 - 2e).$$